Meena

UNIVERSITY OF JAFFNA

Professor
Sivapathasuntharam Mageswaran
Memorial Oration

CHAOS:
THE MIDDLE KINGDOM

by
Professor Ramakrishna Ramaswamy
Vice-Chancellor, University of Hyderabad,
India.

5th June 2012

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SRI LANKA

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Vice-Chancellor's Message

Late Professor Mageswaran was a well recognized and respected academic who contributed immensely to the development of the University of Jaffna, especially to the Faculty of Science. His personal interest in the development of the Faculty of Science and the University as a whole cannot be forgotten. I have witnessed his love towards the University of Jaffna when I was a Senate Member and when the Department of Chemistry was erected.

I am very happy to welcome Professor Ramakrishna Ramaswamy, Vice-Chancellor, University of Hyderabad, India to deliver Professor Mageswaran Memorial oration. Our guest is also an eminent researcher and administrator as well. Being basically a Chemist his interest has been diverted and has inclined towards Physics and at the moment his interests are in Nonlinear Science and Computational Biology. We are very happy and proud to have Professor Ramaswamy to deliver the Memorial Oration on 'Chaos: The Middle Kingdom'.

Professor (Ms.) V.Arasaratnam
Vice-Chancellor
University of Jaffna
05-06-2012

CHAOS: THE MIDDLE KINGDOM

Ramakrishna Ramaswamy

Vice-Chancellor, University of Hyderabad, Hyderabad, India

Vice-Chancellor of the University of Jaffna, Dean of the Faculty of Science, members of the family of Late Professor Mageswaran, distinguished Professors, respected members of the staff of the University, well-wishers and dear students.

I am indeed very pleased to deliver the Professor Mageswaran Memorial Oration for the year 2012, and to honour a person who did so much for the growth of the subject of Chemistry in the University, and thereby in the country. As founder Head of the Department of Chemistry and twice elected Dean of the Faculty of Science, he played a crucial role in shaping this University during its initial years. He served in this University in various capacities during some of the most difficult periods and, despite several obstacles, worked hard to maintain very high standards in its academic and administrative affairs.

Like Professor Mageswaran, I was trained in the subject of Chemistry during my undergraduate and postgraduate years. Though I have moved away and got interested in other fields like Nonlinear Science and Computational Biology, I believe that a love for the subject of Chemistry forges an intellectual bond between us. I understand that Professor Mageswaran possessed natural talents in several fields of knowledge, and I hope that my talk today will be a fitting tribute to him.

In the last few decades there has been considerable progress in understanding the origin of irregular behaviour and its consequences both in space and in time. The natural world has numerous examples of phenomena that vary with time. Some of them are very regular – for instance, each day has precisely 24 hours. The time taken by our earth to revolve around the sun, namely a year, is constant. For most people with healthy hearts, the heart beats in a regular and periodic manner. At the same time we are also surrounded by phenomena that are irregular in time. The weather is a prime example of irregular variation, the annual rainfall being another. The electrical activity in the brain- EEG signals for instance.

What makes these phenomena intrinsically different? What are the physical (or perhaps mathematical) conditions that are either necessary or sufficient for a given system to show either regular or irregular motion? Further, what is the importance of such behaviour? Clearly it is very important that length of a day be constant; if it weren't, a whole host of biological processes would run amok, making life very very difficult. At the same time, the small fluctuations in daytime temperature (another example of irregular variation) are probably not important or consequential.

Such issues have arisen in all fields of science. A famous question posed by the King of Sweden in the 1870's, namely "is the solar system stable?" gave rise to several important developments in the fields of mathematics and physics. (The answer, in brief, is that it depends on where one is located. Some planets, like the earth, will go around the sun in stable orbits, but others would be in unstable orbits that would eventually leave the solar system. These unstable orbits are also irregular, and any planet on such an orbit has long since departed. Something similar happens in the Saturn planetary system, and the unstable orbits are seen as "gaps" in the rings.)

In mathematics, one of the most important open problems is the Riemann hypothesis, namely the conjecture by Riemann that the positions s of the zeros of the zeta function, that is those points where the zeta function vanishes have $\mathrm{Re}(s) = 1/2$. It is known that there are an infinite number of such zeros, and there is an intimate connection between the gaps between the zeros and an unrelated question that is important in nuclear science: how are the energy levels of complex nuclei distributed? Or to give an example from molecular physics, how are the spacings between the vibrational levels of complex molecules distributed? Even closer to the palpable world, we know that one of the leading causes of death is what is loosely termed as a 'heart attack'. The healthy heart is an organ that pumps blood periodically and for the most part beats regularly. When it starts varying in an irregular manner, the phenomenon is termed cardiac arrhythmia, namely, the dynamics of the heart goes from being periodic to aperiodic. Similar examples can be drawn from numerous other fields. Indeed, in every field, the contrast between order and disorder is often a very important subject of study.

This is true at both macroscopic as well as microscopic levels. The fact that there are fundamental irregularities in the physical world is, loosely speaking, implicit in quantum phenomena: an important foundational principle is that of uncertainty. That there could be a lack of precision in simultaneously measuring specific pairs of observables was not something that was palatable to all, and in particular to Einstein who said "Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one'. I, at any rate, am convinced that He does not throw dice."

In the last few decades it has become amply clear that there can be considerable uncertainty in a range of temporal phenomena. Even the simplest of systems can behave in a manner that is 'as random as a coin-toss', that paradigm of uncertain dynamics. Many issues of irregular time variation appear to find some resolution in what is termed chaos theory, namely the study of nonlinear dynamical systems. Chaotic dynamics, in this sense, lies at the intersection of the current understanding of complexity, randomness and unpredictability, and is thus *the middle kingdom*.

Because this dynamics occurs in deterministic systems with no external uncertainty or noise, chaos theory has also - in the sense of Thomas Kuhn – been seen as the harbinger of scientific revolution. There is a paradigm shift in the manner in which randomness is seen as arising, in that it is now believed to be intrinsic to certain types of nonlinear systems which can show behaviour that is 'as random as a coin toss'. This has also captured the public imagination, with a number of books, movies, images and motifs that point to the subject's contemporaneity.

At the heart of the matter is a seemingly simple question: Why are some things difficult to predict, and some things easy? The contrast, for instance, between natural phenomena such as sunrise times or eclipses with the weather provides familiar examples. It will hardly surprise anyone here that there is a website where by entering the latitude and longitude of a place, one can obtain sunrise and sunset times for any day in history. (Entering the data for Jaffna, 9° 45′ N, 80°43′ E, I find that the sun rose today at 5: 46 am and will set at 6:26 pm.) The weather forecast, on the other hand, has been far less precise.

Similarly, some years ago, on the 22nd July 2009, there was a total solar eclipse that was visible in India. In planning to go see the eclipse, I researched as much as I could on the internet and discovered that decades earlier it had been predicted that this eclipse would occur at a specific time and would last for a specific period. There was so much detail available that I was able to plan to which city I would travel to view the eclipse. In the event, the sky was overcast, so that for all the precision with which the eclipse time was predicted, the weather was resistant to such prediction.

The question really is whether there is something intrinsic to the nature of the system to make it either inherently predicable or inherently unpredictable. The importance of nonlinearity has become more widely appreciated, especially since this can fundamentally alter behaviour.

Most of the familiar physical laws are linear, and are approximate representations of the real nonlinear world. These linear laws are valid in restricted regimes, and one of the reasons for this is that linear systems can be analysed completely. Take Hooke's Law - something that is learned in high school- that when a spring is extended, the restoring force is proportional to the extension. Of course we realize that the law is valid only when the extension is small: if you stretch the spring too much, both the law and the spring will break. Or the simple pendulum, where the period is independent of the amplitude of oscillations for small oscillations. This was, of course, crucial for the development of time-keeping: the system is linear for small oscillations, and the solution is a familiar one.

All this breaks down in the nonlinear world. In 1963, while trying to studying flow in the upper atmosphere, the MIT meteorologist Lorenz was led to the study of a seemingly simple set of differential equations, and since there was no analytic solution, he needed to solve these on a computer. This was at a time when computing was in its infancy- and given what was available, he needed to write down the solution of the equations at the end of the working day and start from that point the next day. It is part of the lore of the field – the serendipititious nature of the discovery- that calculations starting from the same point on two different days did not agree. It was Lorenz' great insight that the root cause of this lack of consistency was that he had only specified the starting point to four digit accuracy, and the small difference caused the two solutions to become very different very soon. This property, now termed sensitivity to initial conditions is what is called *chaos*. The experimental growth of small differences makes predictability very difficult.

For such behaviour to be seen, it is essential that the system be nonlinear- and more: not every nonlinear system needs to have this feature, a major example of this being the gravitational two-body problem wherein the orbits are always elliptical. In the years since the discovery of chaos, there have been a number of crucial developments that have depended on both analytic and numerical discoveries.

Although precise prediction is impossible for nonlinear systems in the long run, yet different starting points can end up in very similar states. This is the idea that the dynamics has "attractors", behaviour that is always achieved eventually- as for instance when one takes a pendulum in a room, regardless of how you start it off, it will eventually come to rest. However when one adds the notion of sensitivity to initial conditions to the motion on an attractor, one gets the very important notion of a *strange attractor*, a dynamics in nonlinear system that is inevitable and yet unpredictable in its details.

These notions have also been made quantitative. The distance between two initial conditions grows at an exponential rate that is termed the Lyapunov exponent, λ. The inverse of this exponent gives a characteristic timescale, the Lyapunov time. (For instance, the distance doubles in time $\ln 2/\lambda$, a formula similar to the half-life in radioactive decay.) In any case, any positive exponent will ensure that the dynamics will be chaotic.

Such chaos underlies the weather system, which is one reason why weather prediction is so difficult, especially in the long term. As we all realize, this is a matter of great importance in a variety of fields- travel and transportation, agriculture, and so on, so there is a considerable effort across the world for weather and climate prediction. The best available methods today recognize that there is chaos and use a large number of data points to interpolate information.

But there are many many examples that one can draw upon, and one that is particularly appropriate to cite in a Chemistry department is the that of oscillatory chemical reactions. One of the most famous is the Belusov-Zabhotinsky reaction that involves the oxidation of malonic acid using bromate ion and cerium. This is a very nice demonstration experiment since by using the ferroin indicator, one can see the solution change from blue to red and back. In the last few decades, this has become a paradigm example of a reaction that can be made to operate in both periodic and chaotic patterns by varying concentrations of the components.

The general question that arises therefore is a Shakespearean one: Are systems born chaotic, do they achieve chaoticity, or do they have chaos thrust upon them? An important discovery that occurred in the 1960's and '70's is that many systems that are nonlinear become chaotic as parameters are varied. The changes happen gradually in the sense that the behaviour changes gradually as a function of the parameters, but at special points, there is an intrinsic change of the dynamics, and these are turned bifurcations. There are special routes to chaos as control parameters, namely those that can be varied externally in an experiment are changed. Such behaviour has been known for a long in mathematical equations, one of the simplest being the so-called logistic equation which was introduced in the middle 1800's as a model for population growth, with terms corresponding to both birth as well as death.

The behaviour of this system - which can be easily examined by anyone with a simple computer- is quite dramatic. The logistic map is an iteration, $x \rightarrow rx(1-x)$, which means that one starts with some x_0, $x_1 = r\,x_0\,(1-x_0)$, and so on. Since this is population equation, the variable x is necessarily positive, and is constrained to lie between 0 and 1. The parameter r is also constrained to be positive and less than 4. When r is small, say less than 3, then the dynamics has the following behavior for any initial x_0: as one iterates, one eventually goes to a fixed value that depends on r. This is a stable solution, showing no sensitivity to initial conditions. Above $r = 3$, the iterates eventually oscillate between two values. Above some other value of r iterates will oscillate between 4 values, then between 8, and so on. Thus period-1 orbits go to period-2 , then to period-4, then to 8 and so on to infinity. A period that is infinitely long essentially means that the motion does not repeat: it is *chaotic.*

What is most important is that this behaviour is universal: the dynamics is qualitatively similar for a large class of systems, and quantitatively identical for the same degree of nonlinearity. Furthermore this can be verified both qualitatively and as well as quantitatively in a large number of experimental systems, ranging from electronic circuits to physical oscillators, lasers, chemical reactions, fluid flows, physiological systems, and so on.

Given this universality, and the essentiality of nonlinearity, it is inevitable that there have been a number of different areas where these concepts have been applied. In the reminder of this talk I will discuss a few examples and applications of these ideas.

Synchronization is one of the most important -and most common- forms of temporal organization. This was first noted by Huygens in the 1600s in the motion of two pendulums, and he ascribed the behaviour, namely that independent (linear) systems could, with weak coupling, "oscillate in sympathy". Nonlinear systems also show oscillations, but these are chaotic, namely they have sensitivity to initial conditions. A big surprise that came in the 1990's was the discovery that chaotic systems can *also* synchronize. If you feed the dynamics from one chaotic system into another, it turns out that the dynamics of both systems becomes identical, even they individually show sensitivity to initial conditions. The synchronization of signals is easy to detect because they eventually coincide. One application that has been thought of for long is how such chaotic synchronization can be used to mask information- namely in secure communication.

An important area of application for this idea is in biology where time-keeping is crucial at various levels. A number of biological processes show oscillations- the most familiar being cardiac rhythms and circadian rhythms, but there are also a host of phenomena which are periodic- and in a few cases, necessarily aperiodic. A recent area, systems biology, is largely devoted to the study of biological dynamics and their interactions, although the fact that these are "nano" systems with a low number of molecules means that statistical fluctuations can be large and the systems are noisy.

Other important areas such as chaos control, chaos computation, and quantum chaos have seen significant applications in the past few decades as well. All these assume importance because, as has been emphasized, most systems are nonlinear. And they can show chaotic dynamics, namely sensitivity to initial condition. The consequence being that precise prediction becomes difficult, and so other questions need to be asked, such as the importance of a statistical approach.

Why do we need to study it? Because this provides better understanding of the physical world, because it is there. Paradigm shift or not, chaos theory has changed the way in which many people view phenomena in their chosen fields of study, from physics to biology, to medicine, and even to sociology.

In the early 1960s Eugene Wigner, in a remarkably influential essay titled **The Unreasonable Effectiveness of Mathematics in the Natural Sciences** said "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning. "

Chaos theory provides a splendid example of this appropriateness.

After obtaining his doctoral and postdoctoral training in two of the world's premier institutions, namely the Princeton University and the California Institute of Technology in the USA, Professor Ramaswamy returned to India to join the Tata Institute of Fundamental Research, Bombay in 1981. He later moved to the School of Physical Sciences in the Jawaharlal Nehru University (JNU), New Delhi and became the Professor of Physics in 1990.

When the School of Computational and Integrative Sciences (formerly the School of Information Technology) was started in 2002 in the Jawaharlal Nehru University, Professor Ramaswamy was bestowed a rare honour by being appointed as Professor in the Center for Computational Biology and Bioinformatics in addition to his position in the School of Physical Sciences.

Since 2011 Professor Ramaswamy has been serving as the Vice-Chancellor of the University of Hyderabad.

He has published more than 150 journal and book articles independently and jointly and edited 5 books and conference proceedings. He has served on the editorial board of several journals and has brought out 12 titles in his capacity as the Managing Editor of the series 'Texts and Readings in the Physical Sciences'.

An academic of rare distinction and remarkable personality, Professor Ramaswamy has lectured throughout the world, held visiting positions in some of the most prestigious Universities and is a recipient of several awards and honours including elected fellowships to the Indian National Science Academy and TWAS the Academy of Sciences for the Developing World.

Harikanan, Jaffna.