Inverse Modelling for Estimation of Average Grain Size and Material Constants - An Optimization Approach

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Abstract—**The goal is to build up an inverse model capable of finding the average grain size history during an extrusion process and other material constants by using simulated strain and temperature values. This problem of finding the parameter values is based on linear and nonlinear least-squares regressions, coupled with microstructure control models and the solution of a finite element extrusion model. The problem is ill posed and we use the Tikhonov's regularisation to stabilise the solution process. Further some of the parameters in the model are linear and some are nonlinear. We determine the linear parameters using simple linear algebra and for the computation of non-linear parameters we use *MATLAB*'s routine *lsqnonlin*.**

Keywords: Extrusion; Inverse problem; Parameter estimation; Optimization.

1 Introduction

Numerical modelling of extrusion may be used during a design process to assess the extruded material properties. The extrusion simulation describes the material flow through the die. An extrusion model, which is capable of describing the behaviour of material flow, requires the following input data: (a) material data such as Young's modulus, coefficient of expansion, Poisson's ratio, inelastic heat fraction, specific heat, density, conductivity, flow stress-strain relationship, (b) die design variables and (c) process variables such as ram speed, initial temperature and friction factors.

In reality, material data can be measured using available measuring instruments, but the optimal values of die design variables and process variables are often unknown. In a previous paper[3], we formulated a non-linear least squares inverse model in which the optimal values of die design variables and process variables were estimated to achieve a certain grain size. In the approach the following average grain size equation [7],[8]

$$d = \alpha \left(\frac{d\varepsilon}{dt} \exp\left(\frac{Q}{RT} \right) \right)^{\beta} \tag{1}$$

was used as optimizing criteria to terminate the problem. In equation (1), Q is an activation energy, d is the average recrystallized grain size, T is the temperature, $\frac{d\varepsilon}{dt}$ is the strain rate and R is the universal gas constant.

The values of constants α and β are based on experimental observations and therefore the uncertainty of the constants is very high. Small changes in these values can cause variation in the grain size estimation. The successful application of equation (1) depends on the accuracy of the parameters. Methods to identify the optimal values of α and β are therefore an important part of modelling extrusion processes to increase the reliability of the numerical simulation.

In the present work an alternative method is proposed. The idea is to develop an inverse model for estimating grain size history, using inverse modelling techniques available in the literature. Inverse modelling avoids the use of α and β for the grain size history estimation. To do so we consider the problem in which the material properties of the billet as well as process and die design parameters are known but the material constants α, β and grain size d are not known. The accuracy of the model is examined by using simulated temperature and strain data (generated by the forward model) to which normally-distributed relative noise has been added. We design the inverse model as a least squares minimisation problem associated with an ABAQUS finite element solution of an extrusion process. This is an ill-posed problem and we solve it using regularisation methods.

2 Forward Problem

The thermo-mechanical behaviour of the extrusion process can be described mathematically using conservation of mass, momentum and energy as follows [4], [5], [6]: The mass conservation is

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{V} = 0 \tag{2}$$

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