Results in Physics 43 (2022) 106083

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

New solitary wave solutions and stability analysis for the generalized (3 + 1)-dimensional nonlinear wave equation in liquid with gas bubbles

Yun-Hui Zhao [a], Thilagarajah Mathanaranjan [b], Hadi Rezazadeh [c], Lanre Akinyemi [d], Mustafa Inc [e,f,*]

[a] School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo 454000, China
[b] Department of Mathematics and Statistics, University of Jaffna, Sri Lanka
[c] Faculty of Engineering Technology, Amol University of Special Modern Technologies, Amol, Iran
[d] Department of Mathematics, Lafayette College, Easton, PA, USA
[e] Department of Mathematics, Firat University, 23119 Elazig, Turkey
[f] Department of Medical Research, China Medical University, 40402, Taichung, Taiwan

ARTICLE INFO

Keywords:
Generalized (3 + 1)-dimensional NLWE
New GERFM
Solitary wave solutions
Modulational instability analysis

ABSTRACT

In this study, solitary wave solutions for the generalized (3 + 1)-dimensional nonlinear wave equation (NLWE) are extracted using a new generalized exponential rational function method (GERFM). Numerous nonlinear behaviors in liquids with gas bubbles are described by this equation. The suggested method is used to derive the various kinds of new accurate soliton solutions for the equation. Also, the physical interpretations of some obtained solutions are represented. In addition, a modulational instability analysis framework is used to look at the system's stability.

Introduction

In this paper, we focus on a (3 + 1)-dimensional NLWE in liquid containing gas bubbles [1]

$$\left(u_t + a_1 u u_x + a_2 u_{xxx} + a_3 u_x\right)_x + a_4 u_{yy} + a_5 u_{zz} = 0, \tag{1}$$

$a_i (i = 1, 2, ..5)$ is an arbitrary constant, and $u = u(x, y, z, t)$ is a differentiable function with the spatial coordinates x, y, z and t. Some nonlinear physical processes in a liquid containing a gas bubble can be described by this equation. Its bilinear equation, Bäcklund transformation, infinite conservation laws and N-soliton solutions were systematically structured based on the binary Bell polynomials in [1]. With the aid of the symbolic computation, the mixed lump-stripe solutions, mixed rogue wave-stripe solutions and lump solutions of Eq. (1) were obtained in [2].

To fill the gap of the previous findings, we are motivated to find soliton solutions of the considered model via a recently developed method known as the GERFM. The soliton solutions for NLWE play a significant role in soliton theory, physical sciences and engineering. So the finding of exact soliton solutions for NLWE is very important for mathematicians and physicists [3–9]. To the best of our knowledge, such kinds of abundance of solutions have never been established in earlier literature. The goal of the current research is to determine how to use the new GERFM [10,11] to acquire the solitary wave solutions to the Eq. (1). Additionally, the typical linear-stability analysis is used to study the modulation instability analysis of the stationary solution of this system. With the use of alternative methodologies, analytical solutions are discovered for a number of integer and fractal order models aside from the GERFM [12–33]. However, the suggested approaches are strong instruments for generating the precise solutions of nonlinear differential equations and have attracted a lot of interest recently [34–36].

Description of the method

We provide a quick overview of the new GERFM [10,11] in this section. We first take into account the subsequent NLPDE:

$$N\left(\psi, \psi_x, \psi_{xx}, \psi_{yyx}, \dots\right) = 0\,. \tag{2}$$

If we use $\psi = \psi(\xi)$ and $\xi = k_1 x + k_2 y + k_3 z - \nu t$, then we reduce the NLPDE to the following ODE.

$$N\left(\psi, \psi', \psi'', \dots\right) = 0\,, \tag{3}$$

* Corresponding author at: Department of Mathematics, Firat University, 23119 Elazig, Turkey.
E-mail addresses: zhaoyunhui2019@163.com (Y.-H. Zhao), mathanaranjan@gmail.com (T. Mathanaranjan), h.rezazadeh@ausmt.ac.ir (H. Rezazadeh), akinyeml@lafayette.edu (L. Akinyemi), minc@firat.edu.tr (M. Inc).

https://doi.org/10.1016/j.rinp.2022.106083
Received 15 August 2022; Received in revised form 26 October 2022; Accepted 31 October 2022
Available online 4 November 2022

where k_1, k_2, k_3 and v are constants to be determined. Next, we suppose that the formal solution exists for the Eq. (3)

$$f(\xi) = \frac{p_1 e^{q_1 \xi} + p_2 e^{q_2 \xi}}{p_3 e^{q_3 \xi} + p_4 e^{q_4 \xi}}, \tag{4}$$

where p_1, p_2, p_3, p_4 and q_1, q_2, q_3, q_4 represent the complex/real numbers such that the traveling wave solution of Eq. (2) can be written as follows:

$$\psi(\xi) = A_0 + \sum_{k=1}^{N} A_k f(\xi)^k + \sum_{k=1}^{N} B_k f(\xi)^{-k}. \tag{5}$$

The coefficients $A_0, A_k, B_k\ (1 \le k \le N)$ and $p_n, q_n\ (1 \le n \le 4)$ will be determined such that Eq. (5) satisfies Eq. (3). Note that, the positive integer N can be determined by applying the homogeneous balance principal.

Now, plugging Eq. (5) into (3) and collecting all terms, the following polynomial equation will be acquired:

$$P\left(e^{q_1 \xi}, e^{q_2 \xi}, e^{q_3 \xi}, e^{q_4 \xi}\right) = 0. \tag{6}$$

By setting each coefficient of P to zero, a set of algebraic equation for $p_n, q_n\ (1 \le n \le 4)$ and $k, m, \omega, A_0, A_1, B_1$ will be acquired. Finally, by solving the algebraic equation and then substituting the non-trivial solution in Eq. (5), we can acquire solitons of Eq. (2).

Reduction of the model

To construct the traveling wave solution to Eq. (1), we assume that the wave transformation of the form:

$$u(x,y,z,t) = \phi(\xi), \qquad \xi = k_1 x + k_2 y + k_3 z - vt, \tag{7}$$

where ϕ and ψ are the real function of $\xi = x - \tau t$. Here τ indicates the soliton velocity while ω represents the frequency of the soliton oscillation. Now, by substituting Eq. (7) into Eq. (1), we get,

$$\left(-vk_1 + a_3 k_1^2 + a_4 k_2^2 + a_5 k_3^2\right)\phi''(\xi) + a_1 k_1^2\left(\phi'(\xi)^2 + \phi(\xi)\phi''(\xi)\right) + a_2 k_1^4 \phi^{(4)}(\xi) = 0, \tag{8}$$

Now, integrating Eq. (8) twice and setting the coefficients of integration to zero, we acquire :

$$2\left(-vk_1 + a_3 k_1^2 + a_4 k_2^2 + a_5 k_3^2\right)\phi(\xi) + a_1 k_1^2 \phi(\xi)^2 + 2a_2 k_1^4 \phi''(\xi) = 0. \tag{9}$$

In the next section, we will solve (9) using the proposed method.

Implementation of proposed method

We use the new GERFM [10,11] to find the solitary wave solutions of Eq. (1).

Balancing the $\phi''(\xi)$ and $\phi(\xi)^2$ in (9), yields $N = 2$. According to the new GERFM, we assume that Eq. (9) has the formal solution:

$$\phi(\xi) = A_0 + A_1 f(\xi) + A_2 f(\xi)^2 + \frac{B_1}{f(\xi)} + \frac{B_2}{f(\xi)^2}, \tag{10}$$

where A_i and B_J $(i = 0,1,2 \quad j = 1,2)$ are arbitrary constants to be determined, while the $f(\xi)$ is giving by (4). Substituting (10) into (9) and according the method described above, we acquire the following non-trivial solutions of (1) as follows:

Family 1. If $p = [i, -i, 1, 1]$ and $q = [i, -i, i, -i]$ then the Eq. (4) yields:

$$f(\xi) = -\tan(\xi), \quad \text{where} \quad \xi = k_1 x + k_2 y + k_3 z - vt \tag{11}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{12a_2 k_1^2}{a_1},\ A_2 = -\frac{12a_2 k_1^2}{a_1},\ A_1 = 0,\ B_2 = 0,\ B_1 = 0,$$
$$`v = \frac{a_3 k_1^2 - 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{12}$$

From Eqs. (10) and (11), we acquired:

$$u(x,y,z,t) = -\frac{12a_2 k_1^2}{a_1}\ \sec^2(\xi), \tag{13}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t.$

Case 2.

$$A_0 = -\frac{4a_2 k_1^2}{a_1},\ A_2 = -\frac{12a_2 k_1^2}{a_1},\ A_1 = 0,\ B_2 = 0,\ B_1 = 0,$$
$$v = \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{14}$$

From Eqs. (10) and (11), we acquired:

$$u(x,y,z,t) = \frac{4a_2 k_1^2}{a_1}\ \left[2 - 3\sec^2(\xi)\right], \tag{15}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t.$

Case 3.

$$A_0 = -\frac{12a_2 k_1^2}{a_1},\ A_2 = 0,\ A_1 = 0,\ B_2 = -\frac{12a_2 . k_1^2}{a_1},\ B_1 = 0,$$
$$v = \frac{a_3 k_1^2 - 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{16}$$

From Eqs. (10) and (11), we acquired:

$$u(x,y,z,t) = -\frac{12a_2 k_1^2}{a_1}\ \csc^2(\xi). \tag{17}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t.$

Case 4.

$$A_0 = -\frac{4a_2 k_1^2}{a_1},\ A_2 = 0,\ A_1 = 0,\ B_2 = -\frac{12a_2 k_1^2}{a_1},\ B_1 = 0,$$
$$v = \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{18}$$

From Eqs. (10) and (11), we acquired:

$$u(x,y,z,t) = -\frac{4a_2 k_1^2}{a_1}\ \left[-2 + 3\csc^2(\xi)\right], \tag{19}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t.$

Case 5.

$$A_0 = -\frac{24a_2 k_1^2}{a_1},\ A_1 = 0,\ A_2 = -\frac{12a_2 k_1^2}{a_1},\ B_1 = 0,\ B_2 = -\frac{12a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 - 16a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{20}$$

From Eqs. (10) and (11), we acquired:

$$u(x,y,z,t) = -\frac{48a_2 k_1^2}{a_1}\ \csc^2(2\xi), \tag{21}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - 16a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t.$

Case 6.

$$A_0 = \frac{8a_2 k_1^2}{a_1},\ A_1 = 0,\ A_2 = -\frac{12a_2 k_1^2}{a_1},\ B_1 = 0,\ B_2 = -\frac{12a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 + 16a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{22}$$

From Eqs. (10) and (11), we acquired:

$$u(x,y,z,t) = -\frac{16a_2 k_1^2}{a_1}\left[-2 + 3\csc^2(2\xi)\right], \tag{23}$$

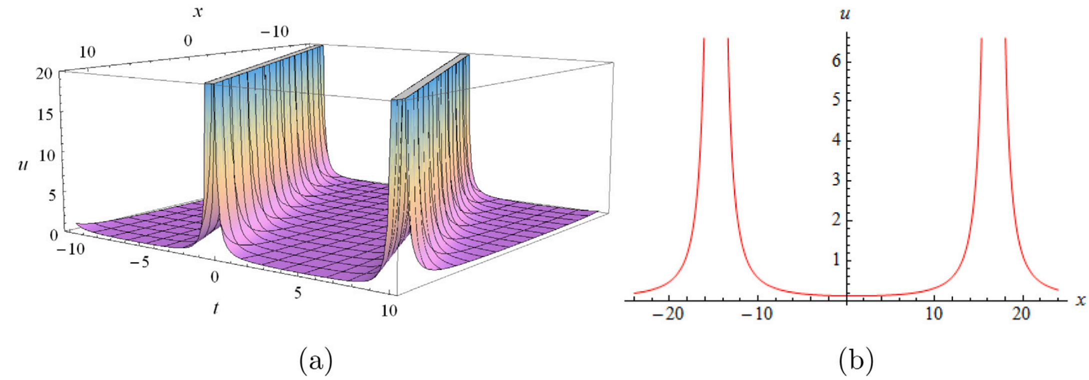

Fig. 1. (a) 3D plot of solution (13) with $k_1 = k_2 = k_3 = 0.1,\ y = z = 1,\ a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

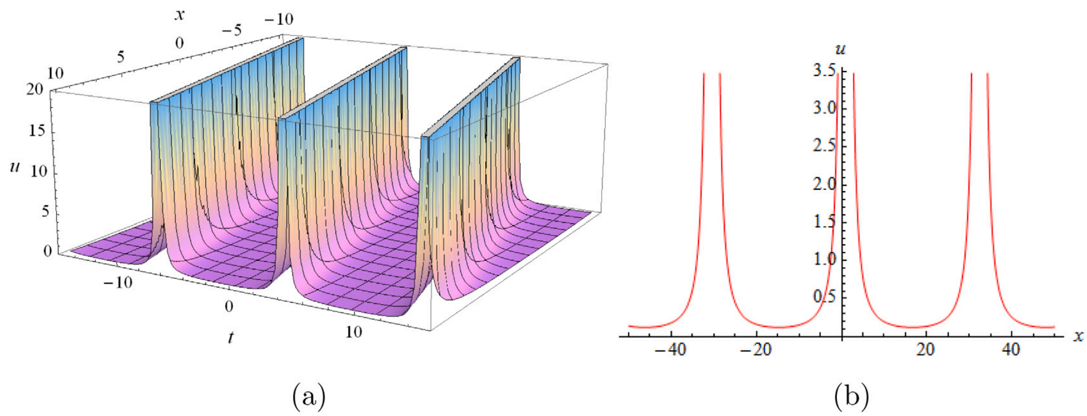

Fig. 2. (a) 3D plot of solution (17) with $k_1 = k_2 = k_3 = 0.1,\ y = z = 1,\ a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2+16a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t.$

Family 2. If $p = [1+i, 1-i, 1, 1]$ and $q = [i, -i, i, -i]$ then the Eq. (4) yields:

$$f(\xi) = 1 - \tan(\xi), \quad \text{where} \quad \xi = k_1x + k_2y + k_3z - vt \tag{24}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{24a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = \frac{48a_2k_1^2}{a_1},\ B_2 = -\frac{48a_2k_1^2}{a_1}, \\ v = \frac{a_3k_1^2 - 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{25}$$

From Eqs. (10) and (24), we acquired:

$$u(x,y,z,t) = \frac{24a_2k_1^2}{a_1[-1+\sin(2\xi)]}. \tag{26}$$

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2-4a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t.$

Case 2.

$$A_0 = -\frac{16a_2k_1^2}{a_1},\ A_2 = 0,\ A_1 = 0,\ B_2 = -\frac{48a_2k_1^2}{a_1},\ B_1 = \frac{48a_2k_1^2}{a_1}, \\ v = \frac{a_3k_1^2 + 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{27}$$

From Eqs. (10) and (24), we acquired:

$$u(x,y,z,t) = -\frac{16a_2k_1^2\left[\sec^2(\xi) + \tan(\xi)\right]}{a_1[-1+\tan(\xi)]^2}. \tag{28}$$

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2+4a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t.$

Family 3. If $p = [1, -1, 1, 1]$ and $q = [-1, 1, -1, 1]$ then the Eq. (4) yields:

$$f(\xi) = -\tanh(\xi), \quad \text{where} \quad \xi = k_1x + k_2y + k_3z - vt \tag{29}$$

The following cases are acquired.

Case 1.

$$A_0 = \frac{4a_2k_1^2}{a_1},\ A_2 = -\frac{12a_2k_1^2}{a_1},\ A_1 = 0,\ B_2 = 0,\ B_1 = 0, \\ v = \frac{a_3k_1^2 - 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{30}$$

From Eqs. (10) and (29), we acquired:

$$u(x,y,z,t) = \frac{4a_2k_1^2}{a_1}\left[1 - 3\tanh^2(\xi)\right], \tag{31}$$

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2-4a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t.$

Case 2.

$$A_0 = \frac{12a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = -\frac{12a_2k_1^2}{a_1},\ B_1 = 0,\ B_2 = 0, \\ v = \frac{a_3k_1^2 + 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{32}$$

From Eqs. (10) and (29), we acquired:

$$u(x,y,z,t) = \frac{12a_2k_1^2}{a_1}\text{sech}^2(\xi), \tag{33}$$

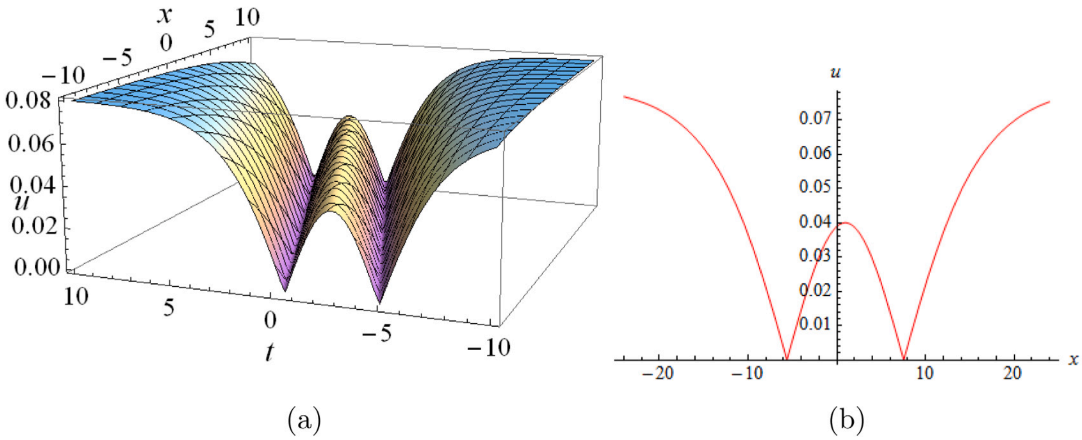

Fig. 3. (a) 3D plot of solution (31) with $k_1 = k_2 = k_3 = 0.1$, $y = z = 1$, $a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

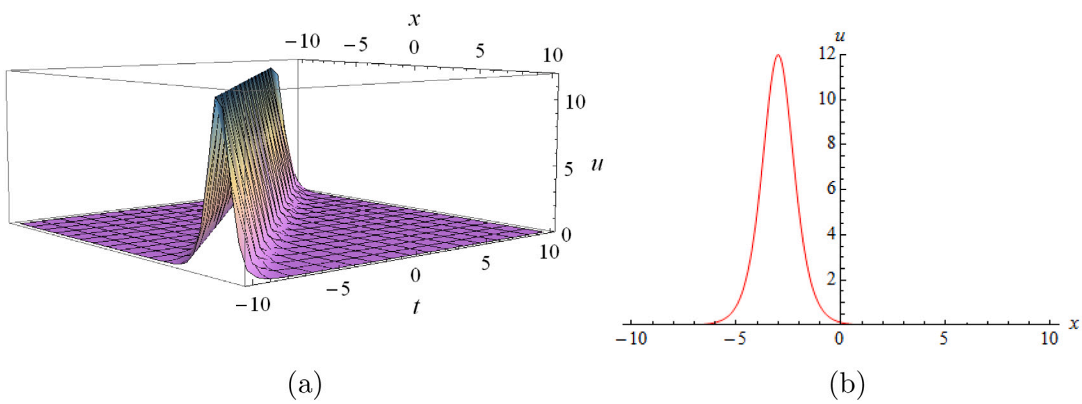

Fig. 4. (a) 3D plot of solution (33) with $k_1 = k_2 = k_3 = 1$, $y = z = 1$, $a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 3.

$$A_0 = \frac{4a_2 k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = 0,\ B_2 = -\frac{12 a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 - 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \quad (34)$$

From Eqs. (10) and (29), we acquired:

$$u(x,y,z,t) = \frac{4a_2 k_1^2}{a_1}\left[1 - 3\coth^2(\xi)\right], \quad (35)$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 4.

$$A_0 = \frac{12a_2 k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = 0,\ B_2 = -\frac{12 a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \quad (36)$$

From Eqs. (10) and (29), we acquired:

$$u(x,y,z,t) = -\frac{12a_2 k_1^2}{a_1}\mathrm{csch}^2(\xi), \quad (37)$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 5.

$$A_0 = -\frac{8a_2 k_1^2}{a_1},\ A_2 = -\frac{12a_2 k_1^2}{a_1},\ A_1 = 0,\ B_2 = -\frac{12 a_2 k_1^2}{a_1},\ B_1 = 0,$$
$$v = \frac{a_3 k_1^2 - 16a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \quad (38)$$

From Eqs. (10) and (29), we acquired:

$$u(x,y,z,t) = -\frac{16a_2 k_1^2}{a_1}\left[2 + 3\mathrm{csch}[2\xi]^2\right], \quad (39)$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - 16a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 6.

$$A_0 = \frac{24a_2 k_1^2}{a_1},\ A_2 = -\frac{12a_2 k_1^2}{a_1},\ A_1 = 0,\ B_2 = -\frac{12 a_2 k_1^2}{a_1},\ B_1 = 0,$$
$$v = \frac{a_3 k_1^2 + 16a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \quad (40)$$

From Eqs. (10) and (29), we acquired:

$$u(x,y,z,t) = -\frac{48a_2 k_1^2}{a_1}\mathrm{csch}^2(2\xi), \quad (41)$$

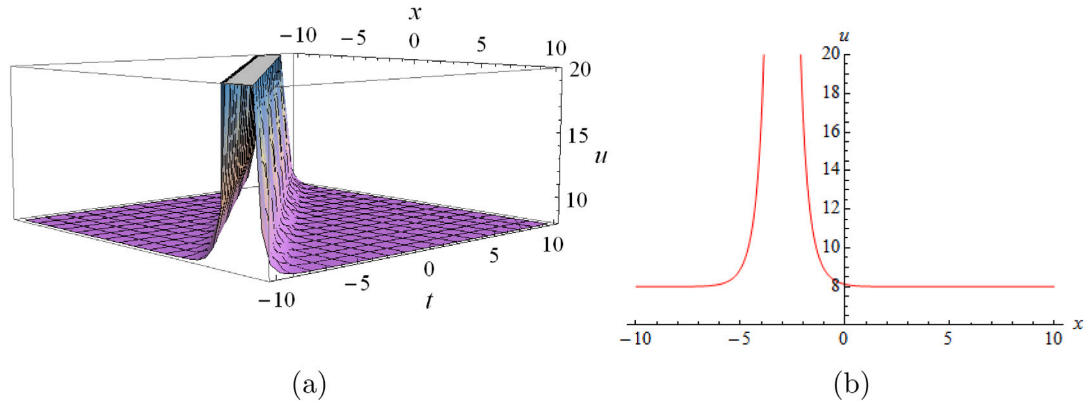

Fig. 5. (a) 3D plot of solution (35) with $k_1 = k_2 = k_3 = 1,\ y = z = 1,\ a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

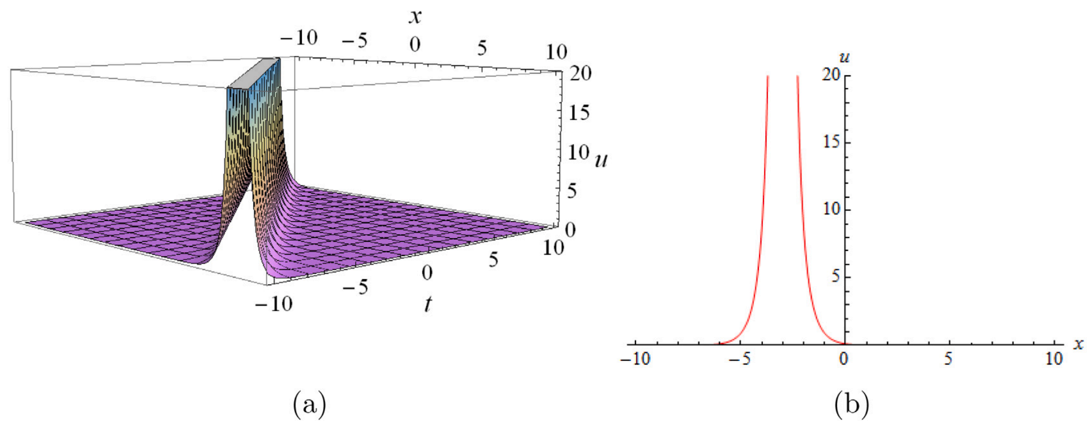

Fig. 6. (a) 3D plot of solution (37) with $k_1 = k_2 = k_3 = 1,\ y = z = 1,\ a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 16 a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}$.

Family 4. If $p = [-2, -3, 1, 1]$ and $q = [1, 0, 1, 0]$ then the Eq. (4) yields:

$$f(\xi) = \frac{-2e^{\xi} - 3}{e^{\xi} + 1}, \quad \text{where} \quad \xi = k_1 x + k_2 y + k_3 z - vt \tag{42}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{74 a_2 k_1^2}{a_1},\ A_2 = 0,\ A_1 = 0,\ B_2 = -\frac{432 a_2 k_1^2}{a_1},\ B_1 = -\frac{360 a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 - a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{43}$$

From Eqs. (10) and (42), we acquired:

$$u(x, y, z, t) = -\frac{2 a_2 k_1^2 \left[9 + 4e^{\xi}\left(-6 + e^{\xi}\right)\right]}{a_1 \left(3 + 2e^{\xi}\right)^2}, \tag{44}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 2.

$$A_0 = -\frac{72 a_2 k_1^2}{a_1},\ A_2 = 0,\ A_1 = 0,\ B_2 = -\frac{432 a_2 k_1^2}{a_1},\ B_1 = -\frac{360 a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 + a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{45}$$

From Eqs. (10) and (42), we acquired:

$$u(x, y, z, t) = \frac{72 a_2 k_1^2 e^{\xi}}{a_1 \left(3 + 2e^{\xi}\right)^2}, \tag{46}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 3.

$$A_0 = -\frac{74 a_2 k_1^2}{a_1},\ A_2 = -\frac{12 a_2 k_1^2}{a_1},\ A_1 = -\frac{60 a_2 k_1^2}{a_1},\ B_2 = 0,\ B_1 = 0,$$
$$v = \frac{a_3 k_1^2 - a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{47}$$

From Eqs. (10) and (42), we acquired:

$$u(x, y, z, t) = -\frac{2 a_2 k_1^2 \left(1 - 4e^{\xi} + e^{2\xi}\right)}{a_1 \left(1 + e^{\xi}\right)^2}, \tag{48}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 4.

$$A_0 = -\frac{72 a_2 k_1^2}{a_1},\ A_1 = -\frac{60 a_2 k_1^2}{a_1},\ A_2 = -\frac{12 a_2 k_1^2}{a_1},\ B_2 = 0,\ B_1 = 0,$$
$$v = \frac{a_3 k_1^2 + a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{49}$$

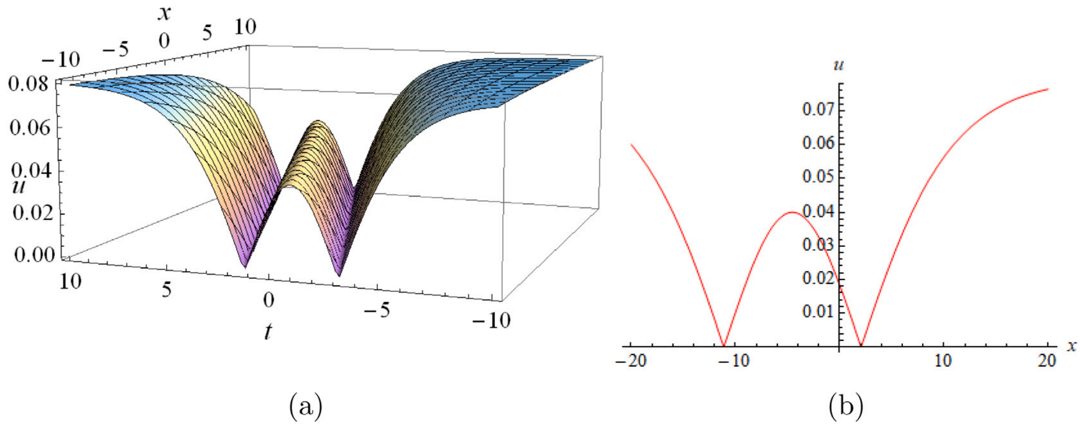

Fig. 7. (a) 3D plot of solution (53) with $k_1 = k_2 = k_3 = 0.1,\ y = z = 1,\ a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

From Eqs. (10) and (42), we acquired:

$$u(x,y,z,t) = \frac{12a_2k_1^2e^{\xi}}{a_1\left(1+e^{\xi}\right)^2}, \tag{50}$$

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2+a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t$.

Family 5. If $p = [-3,-1,1,1]$ and $q = [1,-1,1,-1]$ then the Eq. (4) yields:

$$f(\xi) = -2 - \tanh(\xi), \quad \text{where} \quad \xi = k_1x + k_2y + k_3z - vt \tag{51}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{44a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{144a_2k_1^2}{a_1},\ B_2 = -\frac{108a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 - 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{52}$$

From Eqs. (10) and (51), we acquired:

$$u(x,y,z,t) = -\frac{4a_2k_1^2\left[-1+\tanh(\xi)(8+11\tanh(\xi))\right]}{a_1[2+\tanh(\xi)]^2}, \tag{53}$$

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2-4a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t$.

Case 2.

$$A_0 = -\frac{36a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{144a_2k_1^2}{a_1},\ B_2 = -\frac{108a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 + 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{54}$$

From Eqs. (10) and (51), we acquired:

$$u(x,y,z,t) = \frac{36a_2k_1^2}{a_1[2\cosh(\xi)+\sinh(\xi)]^2}, \tag{55}$$

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2+4a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t$.

Family 6. If $p = [-1,3,1,-1]$ and $q = [1,-1,1,-1]$ then the Eq. (4) yields:

$$f(\xi) = \coth(\xi) - 2, \quad \text{where} \quad \xi = k_1x + k_2y + k_3z - vt \tag{56}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{44a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{144a_2k_1^2}{a_1},\ B_2 = -\frac{108a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 - 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{57}$$

From Eqs. (10) and (56), we acquired:

$$u(x,y,z,t) = \frac{4a_2k_1^2\left[1+(8-11\coth(\xi))\coth(\xi)\right]}{a_1[-2+\coth(\xi)]^2}, \tag{58}$$

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2-4a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t$.

Case 2.

$$A_0 = -\frac{36a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{144a_2k_1^2}{a_1},\ B_2 = -\frac{108a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 + 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{59}$$

From Eqs. (10) and (56), we acquired:

$$u(x,y,z,t) = -\frac{36a_2k_1^2}{a_1[\cosh(\xi)-2\sinh(\xi)]^2}, \tag{60}$$

where $\xi = k_1x + k_2y + k_3z - \frac{a_3k_1^2+4a_2k_1^4+a_4k_2^2+a_5k_3^2}{k_1}t$.

Family 7. If $p = [-2,-1,1,1]$ and $q = [0,1,0,1]$ then the Eq. (4) yields:

$$f(\xi) = \frac{-e^{\xi}-2}{1+e^{\xi}}, \quad \text{where} \quad \xi = k_1x + k_2y + k_3z - vt \tag{61}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{26a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{72a_2k_1^2}{a_1},\ B_2 = -\frac{48a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 - a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{62}$$

From Eqs. (10) and (61), we acquired:

$$u(x,y,z,t) = -\frac{2a_2k_1^2\left[4+e^{\xi}\left(-8+e^{\xi}\right)\right]}{a_1\left(2+e^{\xi}\right)^2}, \tag{63}$$

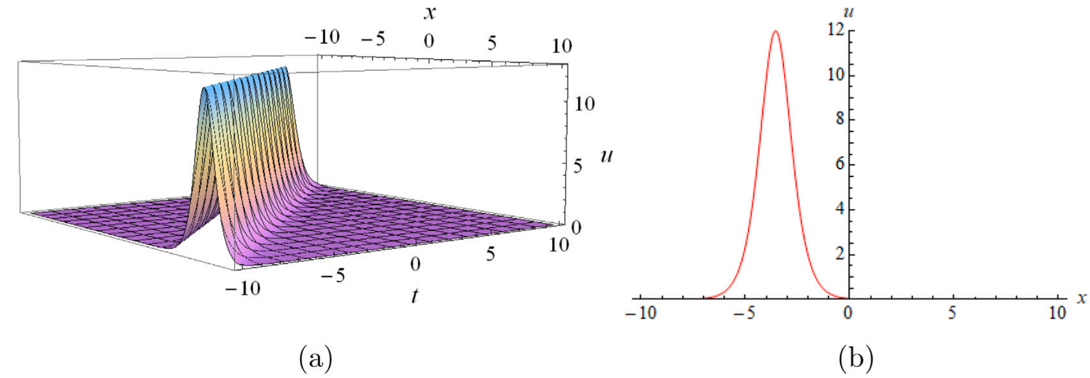

Fig. 8. (a) 3D plot of solution (55) with $k_1 = k_2 = k_3 = 1$, $y = z = 1$, $a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

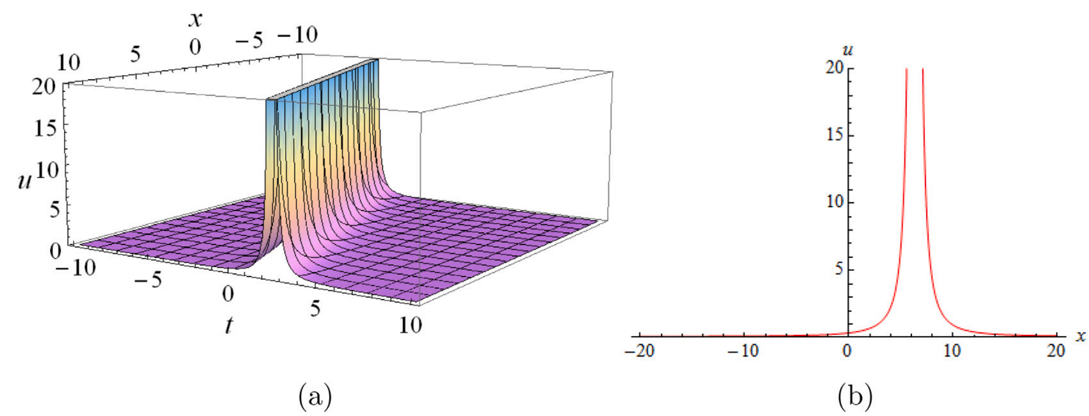

Fig. 9. (a) 3D plot of solution (58) with $k_1 = k_2 = k_3 = 0.1$, $y = z = 1$, $a_1 = a_2 = a_3 = a_4 = a_5 = 1$ (b) Corresponding 2D plot for $t = 1$.

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 2.

$$A_0 = -\frac{24 a_2 k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{72 a_2 k_1^2}{a_1},\ B_2 = -\frac{48 a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 + a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{64}$$

From Eqs. (10) and (61), we acquired:

$$u(x, y, z, t) = \frac{24 a_2 k_1^2 e^{\xi}}{a_1 \left(2 + e^{\xi}\right)^2}, \tag{65}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Family 8. If $p = [2 - i, 2 + i, 1, 1]$ and $q = [i, -i, i, -i]$ then the Eq. (4) yields:

$$f(\xi) = \tan(\xi) + 2, \quad \text{where} \quad \xi = k_1 x + k_2 y + k_3 z - vt \tag{66}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{60 a_2 k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = \frac{240 a_2 k_1^2}{a_1},\ B_2 = -\frac{300 a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 - 4 a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{67}$$

From Eqs. (10) and (66), we acquired:

$$u(x, y, z, t) = -\frac{60 a_2 k_1^2}{a_1 [2\cos(\xi) + \sin(\xi)]^2}, \tag{68}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - 4 a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 2.

$$A_0 = -\frac{52 a_2 k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = \frac{240 a_2 k_1^2}{a_1},\ B_2 = -\frac{300 a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 + 4 a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{69}$$

From Eqs. (10) and (66), we acquired:

$$u(x, y, z, t) = -\frac{4 a_2 k_1^2 \left[7 + \tan(\xi)(-8 + 13\tan(\xi))\right]}{a_1 [2 + \tan(\xi)]^2}, \tag{70}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 4 a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Family 9. If $p = [2, 0, 1, -1]$ and $q = [1, 0, i, -i]$ then the Eq. (4) yields:

$$f(\xi) = -1 + \cot(\xi), \quad \text{where} \quad \xi = k_1 x + k_2 y + k_3 z - vt \tag{71}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{16 a_2 k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{48 a_2 k_1^2}{a_1},\ B_2 = -\frac{48 a_2 k_1^2}{a_1},$$
$$v = \frac{a_3 k_1^2 + 4 a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1}. \tag{72}$$

From Eqs. (10) and (71), we acquired:

$$u(x, y, z, t) = -\frac{16 a_2 k_1^2 \left[\cot(\xi) + \csc(\xi)^2\right]}{\left[-1 + \cot^2(\xi)\right]}, \tag{73}$$

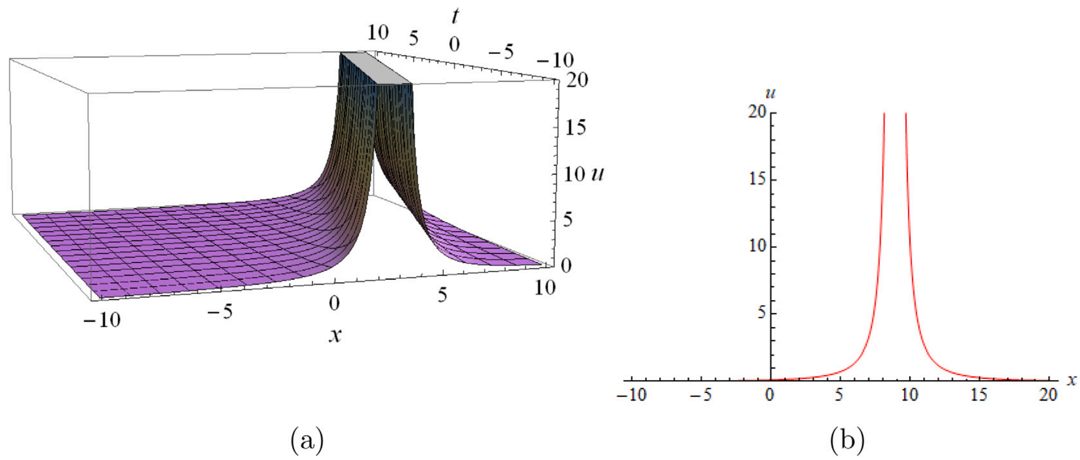

Fig. 10. (a) 3D plot of solution (73) with $k_1 = k_2 = k_3 = 0.1,\ y = z = 1,\ a_1 = a_2 = a_3 = a_4 = a_5 = 0.1$ (b) Corresponding 2D plot for $t = 1$.

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 2.

$$A_0 = -\frac{24a_2k_1^2}{a_1},\ A_1 = -\frac{24a_2k_1^2}{a_1},\ A_2 = -\frac{12a_2k_1^2}{a_1},\ B_1 = 0,\ B_2 = 0,$$
$$v = \frac{a_3k_1^2 - 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{74}$$

From Eqs. (10) and (71), we acquired:

$$u(x,y,z,t) = -\frac{12a_2k_1^2}{a_1}\ \csc^2(\xi), \tag{75}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 3.

$$A_0 = -\frac{16a_2k_1^2}{a_1},\ A_1 = -\frac{24a_2k_1^2}{a_1},\ A_2 = -\frac{12a_2k_1^2}{a_1},\ B_1 = 0,\ B_2 = 0,$$
$$v = \frac{a_3k_1^2 + 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{76}$$

From Eqs. (10) and (71), we acquired:

$$u(x,y,z,t) = -\frac{4a_2k_1^2}{a_1}\ \left[1 + 3\cot^2(\xi)\right], \tag{77}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Family 10. If $p = [-2-i, -2+i, 1, 1]$ and $q = [i, -i, i, -i]$ then the Eq. (4) yields:

$$f(\xi) = \tan(\xi) - 2, \quad \text{where} \quad \xi = k_1 x + k_2 y + k_3 z - vt \tag{78}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{60a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{240a_2k_1^2}{a_1},\ B_2 = -\frac{300a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 - 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{79}$$

From Eqs. (10) and (78), we acquired:

$$u(x,y,z,t) = -\frac{60a_2k_1^2\sec(\xi)^2}{a_1[-2+\tan(\xi)]^2}, \tag{80}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 2.

$$A_0 = -\frac{52a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_1 = -\frac{240a_2k_1^2}{a_1},\ B_2 = -\frac{300a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 + 4a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{81}$$

From Eqs. (10) and (78), we acquired:

$$u(x,y,z,t) = -\frac{4a_2k_1^2\,[7 + \tan(\xi)(8 + 13\tan(\xi))]}{a_1[-2+\tan(\xi)]^2}, \tag{82}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + 4a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Family 11. If $p = [2, 1, 1, 1]$ and $q = [1, 0, 1, 0]$ then the Eq. (4) yields:

$$f(\xi) = \frac{1+2e^{\xi}}{1+e^{\xi}}, \quad \text{where} \quad \xi = k_1 x + k_2 y + k_3 z - vt \tag{83}$$

The following cases are acquired.

Case 1.

$$A_0 = -\frac{26a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_2 = -\frac{48a_2k_1^2}{a_1},\ B_1 = \frac{72a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 - a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{84}$$

From Eqs. (10) and (83), we acquired:

$$u(x,y,z,t) = -\frac{2a_2k_1^2\left[1 + 4e^{\xi}\left(-2 + e^{\xi}\right)\right]}{a_1\left(1+2e^{\xi}\right)^2}, \tag{85}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 - a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

Case 2.

$$A_0 = -\frac{24a_2k_1^2}{a_1},\ A_1 = 0,\ A_2 = 0,\ B_2 = -\frac{48a_2k_1^2}{a_1},\ B_1 = \frac{72a_2k_1^2}{a_1},$$
$$v = \frac{a_3k_1^2 + a_2k_1^4 + a_4k_2^2 + a_5k_3^2}{k_1}. \tag{86}$$

From Eqs. (10) and (83), we acquired:

$$u(x,y,z,t) = \frac{24a_2k_1^2e^{\xi}}{a_1\left(1+2e^{\xi}\right)^2}, \tag{87}$$

where $\xi = k_1 x + k_2 y + k_3 z - \frac{a_3 k_1^2 + a_2 k_1^4 + a_4 k_2^2 + a_5 k_3^2}{k_1} t$.

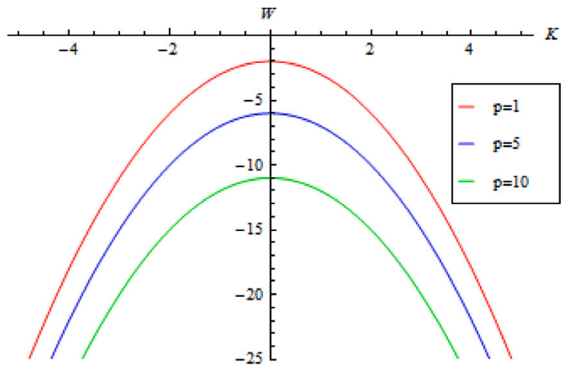

Fig. 11. The dispersion relation between frequency and wave number when $N = M = 1, a_1 = a_2 = a_3 = a_4 = a_5 = 1$.

Linear stability analysis

In section, the linear stability analysis [37,38] will be applied to study the stability analysis for the Eq. (1). The perturbed solution of the considered equation is given by

$$u = p + \lambda q[x, y, z, t], \tag{88}$$

where p is the steady state solution of the Eq. (1). Substituting Eq. (88) into Eq. (1), we get

$$\lambda a_5 q_{zz} + \lambda a_4 q_{zz} + \lambda^2 a_1 q_x^2 + \lambda q_{tx} + p\lambda a_1 q_{xx} + \lambda^2 a_1 q\ q_{xx} + \lambda a_3 q_{xx} + \lambda a_2 q_{xxxx} = 0. \tag{89}$$

By linearization Eq. (89), we get

$$q_{tx} + \left(pa_1 + a_3\right) q_{xx} + a_2 q_{xxxx} + a_4 q_{yy} + a_5 q_{zz} = 0, \tag{90}$$

Suppose that (90) has a solution of the form

$$q = \rho e^{i(tW + Mx + Ny + Kz)}, \tag{91}$$

where M, N, and K are the normalized wave numbers, substituting (91) into (90), then by solving for W, we acquire

$$W = \frac{-M^2 pa_1 + M^4 a_2 - M^2 a_3 - N^2 a_4 - K^2 a_5}{M}, \tag{92}$$

In Fig. 11, the sign of W is negative for all K values. Therefore, the dispersion is stable.

Physical interpretation

In this section, we present some three-dimensional corresponding two-dimensional graphs of the modulus of some of the obtained solutions presented in the previous section. The graphical features of obtained solutions are presented to shed more light on the characteristics of the obtained solutions. From the above Figs. 1–10, one can see that the obtained solutions possess the exact periodic wave solutions and soliton solutions. The construction of the figures is carried out by taking the suitable values of the parameters.

Conclusions

The generalized (3+1)-dimensional NLWE, which describes a variety of nonlinear events in liquid containing gas bubbles, has been taken into consideration in this article. To find fresh soliton solutions for the model under consideration, the new GERFM is used. The stability of the model is examined using the modulation instability analysis. To further illustrate the dynamical behavior of the solutions, the discovered solutions are displayed in 3D and 2D graphs. Furthermore, one future goal is to find possible other soliton solutions of this model by using similar integration techniques.

CRediT authorship contribution statement

Yun-Hui Zhao: Funding acquisition, Conceptualization. **Thilagarajah Mathanaranjan:** Investigation, Writing – review & editing. **Hadi Rezazadeh:** Researcher, Writing – original draft. **Lanre Akinyemi:** Methodology. **Mustafa Inc:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding

Fundamental Research Funds for the Universities of Henan Province (No. NSFRF220420).

Data availability

No data was used for the research described in the article.

References

[1] Tu JM, Tian SF, Xu MJ, Song XQ, Zhang TT. Bäcklund transformation, infinite conservation laws and periodic wave solutions of a generalized (3+1)-dimensional nonlinear wave in liquid with gas bubbles. Nonlinear Dynam 2016;83:1199.
[2] Wang M, Tian B, Sun Y, Zhang Z. Lump, mixed lump-stripe and rogue wave-stripe solutions of a (3+1)-dimensional nonlinear wave equation for a liquid with gas bubbles. Comput Math Appl 2020;79:576–87.
[3] Kaur Lakhveer, Wazwaz Abdul-Majid. Optical soliton solutions of variable coefficient Biswas–Milovic (BM) model comprising Kerr law and damping effect. Optik 2022;266:169617.
[4] Kaur Lakhveer, Wazwaz Abdul-Majid. Dynamical analysis of soliton solutions for space–time fractional Calogero-Degasperis and Sharma-Tasso-Olver equations. Rom Rep Phys 2022;74:108.
[5] Wazwaz Abdul-Majid, Kaur Lakhveer. A new nonlinear integrable fifth-order equation: multiple soliton solutions with unusual phase shifts. Phys Scr 2018;93:115201.
[6] San S, Seadawy AR, Yaşar E. Optical soliton solution analysis for the (2+1) dimensional Kundu–Mukherjee–Naskar model with local fractional derivatives. Opt Quant Electron 2022;54:442.
[7] Biswas Anjan, Yildirim Yakup, Yasar Emrullah, Zhou Qin, Moshokoa Seithuti P, Belic Milivoj. Optical solitons for Lakshmanan–Porsezian–Daniel model by modified simple equation method. Optik 2018;160:24–32.
[8] Yaşar Elif, Yıldırım Yakup, Yaşar Emrullah. New optical solitons of space–time conformable fractional perturbed Gerdjikov-Ivanov equation by sine-Gordon equation method. Results Phys 2018;9:1666–72.
[9] Biswas Anjan, Yildirim Yakup, Yasar Emrullah, Mahmood Mohammad F, Alshomrani Ali Saleh, Zhou Qin, Moshokoa Seithuti P, Belic Milivoj. Optical soliton perturbation for Radhakrishnan–Kundu–Lakshmanan equation with a couple of integration schemes. Optik 2018;163:126–36.
[10] Ghanbari Behzad, Inc Mustafa. A new generalized exponential rational function method to find exact special solutions for the resonance nonlinear Schrödinger equation. Eur Phys J Plus 2018;133:142.
[11] Ghanbari Behzad, Yusuf Abdullahi, Inc Mustafa, Baleanu Dumitru. The new exact solitary wave solutions and stability analysis for the (2 + 1)-dimensional Zakharov-Kuznetsov equation. Adv Difference Equ 2019;49.
[12] Mathanaranjan T. New optical solitons and modulation instability analysis of generalized coupled nonlinear Schrödinger-KdV system. Opt Quantum Electron 2022;54(6):336.
[13] Mathanaranjan T. Exact and explicit traveling wave solutions to the generalized Gardner and BBMB equations with dual high-order nonlinear terms. Partial Differ Equ Appl Math 2021;4:100120.
[14] Senol M, Iyiola OS, Daei Kasmaei H, Akinyemi L. Efficient analytical techniques for solving time-fractional nonlinear coupled Jaulent-Miodek system with energy-dependent Schrödinger potential. Adv Differential Equations 2019;2019:1–21.
[15] Mathanaranjan T, Himalini K. Analytical solutions of the time-fractional nonlinear Schrödinger equation with zero and non zero trapping potential through the Sumudu Decomposition method. J Sci Univ Kelaniya 2019;12:21–33.
[16] Mathanaranjan T, Vijayakumar D. Laplace decomposition method for time-fractional Fornberg-Whitham type equations. J Appl Math Phys 2021;9:260–71.

[17] Akinyemi L, Şenol M, Iyiola OS. Exact solutions of the generalized multidimensional mathematical physics models via sub-equation method. Math Comput Simulation 2021;182:211–33.
[18] Mathanaranjan T. Solitary wave solutions of the Camassa–Holm-Nonlinear Schrödinger equation. Results Phys 2020;19:103549.
[19] Mathanaranjan T. Soliton solutions of deformed nonlinear Schrödinger equations using ansatz method. Int J Appl Comput Math 2021;7:159.
[20] Mathanaranjan T. Optical solitons and stability analysis for the new (3+1)-dimensional nonlinear Schrödinger equation. J Nonlinear Opt Phys Mater 2022. http://dx.doi.org/10.1142/S0218863523500169.
[21] Mathanaranjan T. Optical singular and dark solitons to the (2 + 1)-dimensional time-space fractional nonlinear Schrödinger equation. Results Phys 2021;22:103870.
[22] Kaur Lakhveer, Wazwaz Abdul-Majid. Einstein's vacuum field equation: Painlevé analysis and Lie symmetries. Waves Random Complex Media 2021;31(2):199–206.
[23] Singh Jagdev, Gupta Arpita, Baleanu Dumitru. On the analysis of an analytical approach for fractional Caudrey-Dodd-Gibbon equations. Alex Eng J 2021;61(7):5073–82.
[24] Behzad Ghanbari, Devendra Kumar, Jagdev Singh. Exact solutions of local fractional longitudinal wave equation in a magnetoelectro-elastic circular rod in fractal media. Indian J Phys 2022;96:787–94.
[25] Yadav Supriya, Kumar Devendra, Nisar Kottakkaran Sooppy. A reliable numerical method for solving fractional reaction–diffusion equations. J King Saud Univ-Sci 2021;33(2):101320.
[26] Dubey Ved Prakash, Singh Jagdev, Alshehri Ahmed M, Dubey Sarvesh, Kumar Devendra. An efficient analytical scheme with convergence analysis for computational study of local fractional Schrödinger equations. Math Comput Simulation 2022;196:296–318.
[27] Biswas Anjan, Yıldırım Yakup, Yaşar Emrullah, Zhou Qin, Moshokoa Seithuti P, Belic Milivoj. Sub pico-second pulses in mono-mode optical fibers with Kaup–Newell equation by a couple of integration schemes. Optik 2018;167:121–8.
[28] Nisar KS, Akinyemi L, Inc M, Şenol M, Mirzazadeh M, Houwe A, Rezazadeh H. New perturbed conformable Boussinesq-like equation: Soliton and other solutions. Results Phys 2022;105200.
[29] Younas U, Rezazadeh H, Ren J, Bilal M. Propagation of diverse exact solitary wave solutions in separation phase of iron (Fe-Cr- X (X= Mo, Cu)) for the ternary alloys. Internat J Modern Phys B 2022;36(04):2250039.
[30] Mathanaranjan T, Rezazadeh H, Şenol M, Akinyemi L. Optical singular and dark solitons to the nonlinear Schrödinger equation in magneto-optic waveguides with anti-cubic nonlinearity. Opt Quantum Electron 2021;53(12):1–16.
[31] Mathanaranjan T, Kumar D, Rezazadeh H, Akinyemi L. Optical solitons in metamaterials with third and fourth order dispersions. Opt Quantum Electron 2022;54(5):1–15.
[32] Mathanaranjan T. An effective technique for the conformable space–time fractional cubic-quartic nonlinear Schrodinger equation with different laws of nonlinearity. Comput Methods Differ Equ 2022;10(3):701–15.
[33] Mathanaranjan T, Vijayakumar D. New soliton solutions in nano-fibers with space–time fractional derivatives. 2022, http://dx.doi.org/10.1142/S0218348X22501419.
[34] Kumar S, Niwas M. New optical soliton solutions of Biswas–Arshed equation using the generalised exponential rational function approach and Kudryashov's simplest equation approach. Pramana-J Phys 2022;96:204.
[35] Ali Karmina K, Yokus Asıf, Seadawy Aly R, Yilmazer Resat. The ion sound and langmuir waves dynamical system via computational modified generalized exponential rational function. Chaos Solitons Fractals 2022;161:112381.
[36] Tarla Sibel, Ali1 Karmina K, Yilmazer Resat, Osman MS. On dynamical behavior for optical solitons sustained by the perturbed Chen–Lee–Liu model. Commun Theor Phys 2022;74:075005.
[37] Manafian Jalil, Ilhan Onur Alp, Ismael Hajar Farhan, Mohammed Sizar Abid, Mazanova Saadat. Periodic wave solutions and stability analysis for the (3+1)-D potential-YTSF equation arising in fluid mechanics. Int J Comput Math 2021;98(8):1594–616.
[38] Manafian Jalil, Ilhan Onur Alp, Alizadeh Asad. Periodic wave solutions and stability analysis for the KP-BBM equation with abundant novel interaction solutions. Phys Scr 2020;95:065203, (13pp).