Integral Equation Solution for Microstrip Structures at Low Frequencies

Kandasamy Pirapaharan*[1], Weng Cho Chew[1], and Jun-Sheng Zhao[2]

[1] Center for Computational Electromagnetics and Electromagnetics laboratory
Department of Electrical and Computer Engineering, University of Illinois,
Urbana, Illinois 61801-2991

[2] Madmax Optics, 3035 Whitney Avenue, Hamden, CT 06518

1. Introduction

Microstrip structures have been studied extensively using various types of full wave analysis techniques. However, these techniques are having difficulties because they usually involve the solution of a very large system of linear equations. In this approach, a symmetrical form of electric-field spatial-domain Green's function [1] different from [2] and [3] is applied. Further, the numerical solution of Maxwell's equations at low frequencies is plagued with numerous problems. Because of the discrepant frequency dependence of the solenoidal and irrotational components of the current when the frequency tends to zero, a working numerical method has to include this Helmholtz decomposition and ascribe the requisite frequency dependencies to the solenoidal and irrotational components of the current. This decomposition is achieved by selecting the loop-tree basis [4], [5]. The use of the loop-tree basis, followed by frequency normalization, solves the problem of singular matrices at low frequencies. However, if an iterative solver is used, the iteration count is usually very large and may even diverge for some problems. To overcome this problem, a method of transformation of the matrix equations [6] is also applied.

2. Formulation

For a geometry of a microstrip structure as shown in Fig. 1, the spectral domain Green's function can be derived in a closed form as the sum of TE and TM to z-waves propagating in the positive and negative z directions. After some derivations, the spectral domain dyadic Green's function $\tilde{\overline{\overline{\mathbf{G}}}}$ in the region $z > 0$ can be written in a symmetric form [1] as follows:

$$\begin{aligned}\hat{\alpha}\cdot\tilde{\overline{\overline{\mathbf{G}}}}\cdot\hat{\alpha}' &= \left(\alpha_x\alpha'_x+\alpha_y\alpha'_y\right)\left(\tilde{g}^P+\tilde{g}^{\mathrm{TE},R}\right)+\alpha_z\alpha'_z\left(\tilde{g}^P+\tilde{g}^{\mathrm{TM},R}\right)\\ &\quad+\frac{1}{k^2}\hat{\alpha}\cdot\nabla\nabla\cdot\hat{\alpha}'\,\tilde{g}^P+\frac{1}{k^2}\hat{\alpha}\cdot\nabla\nabla\cdot\hat{\alpha}'_I\tilde{g}^{\mathrm{TM},R}\\ &\quad-\hat{\alpha}\cdot\nabla_s\nabla_s\cdot\hat{\alpha}'_I\tilde{g}^{\mathrm{EM}}\end{aligned} \tag{1}$$

where $\hat{\alpha}=\alpha_x\hat{x}+\alpha_y\hat{y}+\alpha_z\hat{z}$, $\hat{\alpha}'=\alpha'_x\hat{x}+\alpha'_y\hat{y}+\alpha'_z\hat{z}$, $\hat{\alpha}'=-\alpha'_x\hat{x}-\alpha'_y\hat{y}+\alpha'_z\hat{z}$

$$\tilde{g}^P=-\frac{\omega\mu_0}{8\pi^2}\frac{e^{i\mathbf{k}_s\cdot(\mathbf{r}_s-\mathbf{r}'_s)}}{k_z}e^{ik_z|z-z'|}\,,\qquad \tilde{g}^{\mathrm{TM},R}=-\frac{\omega\mu_0}{8\pi^2}\tilde{R}^{\mathrm{TM}}\frac{e^{i\mathbf{k}_s\cdot(\mathbf{r}_s-\mathbf{r}'_s)}}{k_z}e^{ik_z(z+z')}$$

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$$\tilde{g}^{\mathrm{TE},R} = -\frac{\omega\mu_0}{8\pi^2}\tilde{R}^{\mathrm{TE}}\frac{e^{i\mathbf{k}_s\cdot(\mathbf{r}_s-\mathbf{r}'_s)}}{k_z}e^{ik_z(z+z')}, \qquad \tilde{g}^{\mathrm{EM}} = \left(\tilde{g}^{\mathrm{TM},R} + \tilde{g}^{\mathrm{TE},R}\right)/k_s^2$$

k is the wavenumber in free space, $k_s^2 = k_x^2 + k_y^2$ and $\tilde{R}^{\mathrm{TM,TE}}$ is the generalized reflection coefficient of the layered medium. The spectral integration of (1) yields the spatial domain Green's function as follows:

$$\begin{aligned}\hat{\alpha}\cdot\overline{\mathbf{G}}\cdot\hat{\alpha}' &= \left(\alpha_x\alpha'_x + \alpha_y\alpha'_y\right)\left(g^P + g^{\mathrm{TE},R}\right) + \alpha_z\alpha'_z\left(g^P + g^{\mathrm{TM},R}\right) \\ &+ \frac{1}{k^2}\hat{\alpha}\cdot\nabla\nabla\cdot\hat{\alpha}'g^P + \frac{1}{k^2}\hat{\alpha}\cdot\nabla\nabla\cdot\hat{\alpha}'_I g^{\mathrm{TM};R} \\ &- \hat{\alpha}\cdot\nabla_s\nabla_s\cdot\hat{\alpha}'_I g^{\mathrm{EM}}\end{aligned} \tag{2}$$

where $g^\beta = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\tilde{g}^\beta dk_x dk_y \qquad \beta = P, (\mathrm{TE}, R), (\mathrm{TM}, R) \text{ and EM}$

Using the dyadic electric-field Green's function $\overline{\mathbf{G}}$ for the layered medium, an electric-field integral equation (EFIE) can be constructed by enforcing the total electric-field tangential to the surface S to vanish

$$\begin{aligned}&\hat{z}\times\int_S\left(g^P(\mathbf{r},\mathbf{r}') + g^{\mathrm{TE},R}(\mathbf{r},\mathbf{r}')\right)\mathbf{J}(\mathbf{r}')d\mathbf{r}' \\ &+ \hat{z}\times\nabla_s\int_S\left\{\frac{1}{k^2}\left(g^P(\mathbf{r},\mathbf{r}') - g^{\mathrm{TM},R}(\mathbf{r},\mathbf{r}')\right) + g^{\mathrm{EM}}\right\}\nabla'_s\cdot\mathbf{J}(\mathbf{r}')d\mathbf{r}' = -\hat{z}\times\mathbf{E}^{\mathrm{inc}}(\mathbf{r})\end{aligned} \tag{3}$$

where $\mathbf{E}^{\mathrm{inc}}(\mathbf{r})$ can be the field of a impinging plane wave or the field created by a finite source residing within the microstrip structure. Using the loop-tree basis function designed for low-frequency problems,

$$\mathbf{J}(\mathbf{r}') = \sum_{n=1}^{N_L} I_{Ln}\mathbf{J}_{Ln}(\mathbf{r}') + \sum_{n=1}^{N_T} I_{Tn}\mathbf{J}_{Tn}(\mathbf{r}') \tag{4}$$

we discretize the EFIE into a linear algebraic system of equations, where $\mathbf{J}_{Ln}(\mathbf{r}')$ and $\mathbf{J}_{Tn}(\mathbf{r}')$ are the divergence-free surface-loop basis and the nondivergence-free surface-tree basis, respectively. By substituting (4) into (3), testing with $\mathbf{J}_{Lm}(\mathbf{r})$ and $\mathbf{J}_{Tm}(\mathbf{r})$, and applying $\nabla_s\cdot\mathbf{J}_{Lm}(\mathbf{r}) = 0$ and $\nabla_s\cdot\mathbf{J}_{Ln}(\mathbf{r}') = 0$, we simplify to a matrix form as follows:

$$\begin{bmatrix}\overline{\mathbf{Z}}_{LL} & \overline{\mathbf{Z}}_{LT} \\ \overline{\mathbf{Z}}_{TL} & \overline{\mathbf{Z}}_{TT}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{I}_L \\ \mathbf{I}_T\end{bmatrix} = \begin{bmatrix}\mathbf{V}_L \\ \mathbf{V}_T\end{bmatrix} \tag{5}$$

where $\mathbf{V}_L = -\left\langle\mathbf{J}_L(\mathbf{r}), \mathbf{E}^{\mathrm{inc}}(\mathbf{r})\right\rangle$

$\mathbf{V}_T = -\left\langle\mathbf{J}_T(\mathbf{r}), \mathbf{E}^{\mathrm{inc}}(\mathbf{r})\right\rangle$

$\overline{\mathbf{Z}}_{LL} = \left\langle\mathbf{J}_L(\mathbf{r}), g^V(\mathbf{r},\mathbf{r}'), \mathbf{J}_L^t(\mathbf{r}')\right\rangle$

$\overline{\mathbf{Z}}_{LT} = \left\langle\mathbf{J}_L(\mathbf{r}), g^V(\mathbf{r},\mathbf{r}'), \mathbf{J}_T^t(\mathbf{r}')\right\rangle$

$\overline{\mathbf{Z}}_{TL} = \left\langle\mathbf{J}_T(\mathbf{r}), g^V(\mathbf{r},\mathbf{r}'), \mathbf{J}_L^t(\mathbf{r}')\right\rangle = \overline{\mathbf{Z}}_{LT}^t$

$\overline{\mathbf{Z}}_{TT} = \left\langle\mathbf{J}_T(\mathbf{r}), g^V(\mathbf{r},\mathbf{r}'), \mathbf{J}_T^t(\mathbf{r}')\right\rangle - \frac{1}{k^2}\left\langle\nabla_s\cdot\mathbf{J}_T(\mathbf{r}), g^S(\mathbf{r},\mathbf{r}'), \nabla'_s\cdot\mathbf{J}_T^t(\mathbf{r}')\right\rangle$

$$g^V(\mathbf{r},\mathbf{r}') = g^P(\mathbf{r},\mathbf{r}') + g^{TE,R}(\mathbf{r},\mathbf{r}')$$
$$g^S(\mathbf{r},\mathbf{r}') = \left(g^P(\mathbf{r},\mathbf{r}') - g^{\mathrm{TM},R}(\mathbf{r},\mathbf{r}') + k^2 g^{\mathrm{EM}}\right)$$
$$\left\langle \mathbf{A}(\mathbf{r}), g(\mathbf{r},\mathbf{r}'), \mathbf{B}'(\mathbf{r}')\right\rangle = \int \mathbf{A}(\mathbf{r})d\mathbf{r} \cdot \int g(\mathbf{r},\mathbf{r}')\mathbf{B}'(\mathbf{r}')d\mathbf{r}'.$$

Also, $\mathbf{J}_L(\mathbf{r}')$, $\mathbf{J}_L(\mathbf{r})$, $\mathbf{I}_L$, $\mathbf{J}_T(\mathbf{r}')$, $\mathbf{J}_T(\mathbf{r})$, and $\mathbf{I}_T$ are column vectors containing $\mathbf{J}_{Ln}(\mathbf{r}')$, $\mathbf{J}_{Lm}(\mathbf{r})$, I_{Ln}, $\mathbf{J}_{Tn}(\mathbf{r}')$, $\mathbf{J}_{Tm}(\mathbf{r})$, and I_{Tn}, respectively.

When $\omega \to 0$ $(k \to 0)$, the matrix equation is unbalanced and ill conditioned. Since the lower-right block of the matrix becomes dominant in an electric field equation, frequency normalization can be used to balance the matrix as below:

$$\begin{bmatrix} \overline{\mathbf{Z}}_{LL}(O(1)) & k\overline{\mathbf{Z}}_{LT}(O(\omega)) \\ k\overline{\mathbf{Z}}_{TL}(O(\omega)) & k^2\overline{\mathbf{Z}}_{TT}(O(1)) \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I}_L(O(1)) \\ \frac{1}{k}\mathbf{I}_T(O(1)) \end{bmatrix} = \begin{bmatrix} \mathbf{V}_L(O(\omega)) \\ k\mathbf{V}_T(O(\omega)) \end{bmatrix} \tag{6}$$

The above matrix equation is balanced and can be solved by the direct inversion method. However, if the matrix equation is solved by iterative solvers, the iteration count is usually very large. Even though the electrostatic part converges very slowly, the magnetostatic part converges rapidly. However, the electrostatic problem based on pulse basis converges rapidly. Therefore, the charge basis arising from the divergence of the current basis is the main reason for the matrix ill conditioning. To avoid this, we transform the electrostatic part in the matrix equation by basis rearrangement so that the resultant matrix reduces to that based on the pulse basis in the static limit as given in [6].

Expanding the surface charge densities in terms of pulse basis $\rho(\mathbf{r}) = \sum_{n=1}^{N_P} Q_n P_n(\mathbf{r})$ and applying the condition for the charge neutral system, we obtain the expression for the surface charge density as follows:

$$\rho(\mathbf{r}) = \sum_{n=1}^{N_P-1}\left[P_n(\mathbf{r}) - C_{nN_P}P_{N_P}(\mathbf{r})\right]Q_n = \mathbf{N}'(\mathbf{r})\cdot\mathbf{Q} \tag{7}$$

where $C_{nN_P} = \int_{S_n} P_n(\mathbf{r})dS \left[\int_{S_{N_P}} P_{N_P}(\mathbf{r})dS\right]^{-1}$, $\mathbf{N}(\mathbf{r})$ and $\mathbf{Q}$ are vectors of length $N_P - 1$.

Using the current continuity condition $\nabla\cdot\mathbf{J}(\mathbf{r}) = i\omega\rho(\mathbf{r})$, applying $\nabla_s\cdot\mathbf{J}_L(\mathbf{r}) = 0$, taking the inner product with $\mathbf{P}(\mathbf{r})$, we have

$$\left\langle \mathbf{P}(\mathbf{r}), \nabla\cdot\mathbf{J}_T'(\mathbf{r})\right\rangle \cdot \mathbf{I}_T = i\omega\left\langle \mathbf{P}(\mathbf{r}), \mathbf{N}'(\mathbf{r})\right\rangle \cdot \mathbf{Q} \tag{8}$$

In this manner, $\left\langle \mathbf{P}(\mathbf{r}), \mathbf{N}'(\mathbf{r})\right\rangle$ is a diagonal matrix and we can rewrite (8) as

$$\overline{\mathbf{K}}\cdot\mathbf{I}_T = i\omega\,\mathbf{Q} \tag{9}$$

where $\overline{\mathbf{K}}$ is a square matrix. Applying (8) in (6), we obtain the transform matrix as below with good spectral property and the matrix equation converges rapidly.

$$\begin{bmatrix} -i\overline{\mathbf{Z}}_{LL} & \omega\overline{\mathbf{Z}}_{LT}\cdot\mathbf{K}^{-1} \\ k^2/\omega\,\overline{\mathbf{K}}'^{-1}\cdot\overline{\mathbf{Z}}_{TL} & ik^2\overline{\mathbf{K}}'^{-1}\cdot\overline{\mathbf{Z}}_{TT}\cdot\mathbf{K}^{-1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I}_L \\ \mathbf{Q} \end{bmatrix} = \begin{bmatrix} -i\mathbf{V}_L \\ k^2/\omega\,\overline{\mathbf{K}}'^{-1}\cdot\mathbf{V}_T \end{bmatrix} \tag{10}$$

3. Numerical results

Current distribution pattern shown in Fig. 2 has been obtained due to a $\hat{\theta}$ polarized plane wave impinging the rectangular surface with the incident angle of $\theta = 60^0$ and $\phi = 0^0$ at the frequency of 1 kHz. The current coefficients have been obtained with 79 iterations for the unknowns of 1160. It is noted that the current distribution pattern satisfies the symmetry on x axis since the incident angle $\phi = 0^0$.

4. Conclusion

A symmetric form of layered medium Green's function is successfully used to analyze the microstrip structure at low frequencies with the help of loop-tree basis functions, frequency normalization as well as the basis rearrangement. It converges fast and no low-frequency break-down occurs in the numerical computations. In the future, it will be capable of solving large scale problems.

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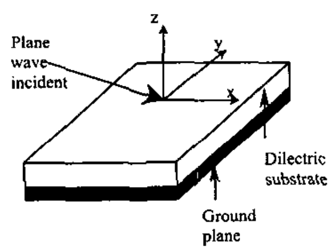

Fig. 1: Microstrip structure

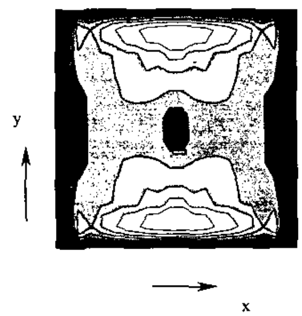

Fig. 2: Current distribution pattern