ABSTRACT

Quaternionic Reproducing Kernel Hilbert Space

Balachandiran Muraleetharan

In this thesis, a set of reproducing kernel Hilbert spaces are obtained on Hilbert spaces over quaternion slices with the aid of coherent states. It is proved that the so obtained set forms a measurable field of Hilbert spaces and their direct integral appears again as a reproducing kernel Hilbert space for a bigger Hilbert space over the whole quaternions. Hilbert spaces over quaternion slices are identified as representation spaces for a set of irreducible unitary group representations and their direct integral is shown to be a reducible representation for the Hilbert space over the whole quaternion field. Then parallel to the quantization of the complex plane, using the canonical coherent states of a right quaternionic Hilbert space, quaternion field of quaternionic quantum mechanics is quantized. Associated upper symbols, lower symbols and related quantities are obtained. It is shown that the right quaternionic canonical coherent states in the Hilbert spaces over quaternion slices saturate the Heisenberg uncertainty relation and thereby they form a set of intelligent states and become a set of minimum uncertainty states. Remarkably using a left multiplication defined on a right quaternionic Hilbert space, linear self-adjoint momentum operators on a right quaternionic Hilbert space are defined in complete analogy with their complex counterpart. With the aid of the so-obtained position and momentum operators, we obtained the Heisenberg uncertainty principle on the whole set of quaternions. For the quaternionic harmonic oscillator, the uncertainty relation is shown to saturate on a neighborhood of the origin in the case we consider the whole set of quaternions. In analogy with the complex Weyl-Heisenberg Lie algebra, Lie algebraic structures are developed for the quaternionic case. We also introduce a quaternionic displacement operator which is square integrable, irreducible and unitary, and we study its properties. Finally, using a left multiplication defined on a right quaternionic Hilbert space, it is demonstrated that pure squeezed states can be defined with all the desired properties on a right quaternionic Hilbert space. Further, we demonstrated that squeezed states can be defined on the same Hilbert space considering the quaternionic slice wise approach and the desired properties are also obtained for quaternionic squeezed states. In conclusion the results furnished in this thesis fulfill the eighty years old conjecture of Birkhoff and von Neumann with the development of almost all the necessary tool for establishing the theory of quaternionic quantum mechanics.

Keywords: Quaternion, Coherent states, Reproducing Kernel, Measurable Field, Lie algebra, Displacement operator

Recommended

Prof. S. Sivathayalan K. Thirulogasanthar