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Two Dimensional Space Curve Based Curvature Induced Stiffness Formulation for Large Deformable Structural Cables

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***Abstract*—In this paper, we present a curvature based modification to the existing beam element formulation to model cables which undergoes large displacements. The proposed approximation is numerically tested against the P-delta formulation and true nonlinear formulation of cables. As a result, a limiting value for the curvature based stiffness is reported.**

***Keywords*—cables, finite elements, small strains, geometric nonlinartity, large displacements.**

I. Introduction

In this paper, we present the numerical study of cable finite element using stiffness matrices obtained form beam and string type elements. Both stiffness matrices include large displacements as a second order effect. In practical point of view, cables are modeled with beam elements with reduced bending stiffness and P-delta effect [11]. Cables are highly flexible structural elements. The deformed shape of cables is load dependent.

In the analysis of cable structures, incremental strain formulation with nonlinear capabilities are demanded [1, 3]. It requires incremental-iterative procedures which requires high computing power and finite element skills. In addition, true nonlinear iso-parametric cable elements with three nodes were developed with non-incremental forms.

Theoretically, cables (such as strings) do not show significant bending moments. The primary load transfer mechanism is through axial deformation that results in heavy axial loads. Therefore, using beam element with P-delta gives limited corrections to the problem considered [10, 7].

This paper examines the limits of using beam element by modifying the bending stiffness into curvature that resulted from the kinematic conditions. This method will release the bending effect and hence include the large displacements as a second order correction.

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II. Kinematics

A. *Displacement field*

Consider a plane cable that undergoes displacements. In this case, assume that material law is linear and small axial strains. It is obvious that cable may undergo large rotations and displacements but the axial strain may be small. Since plane cable is considered, it does not show any torsion in the cable. The deformed location ($\boldsymbol{x}$) of material point is given by the vector sum of original location ($\boldsymbol{X}$) and displacement vector ($\boldsymbol{u}$)

$$\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{u} \tag{1}$$

The unit tangent vector to the deformed configuration is given by the arc-length (s) derivative of the coordinate vector.

$$\boldsymbol{t} = \frac{\partial \boldsymbol{x}}{\partial s} \tag{2}$$

We assume that the normal component of the displacement vector is u_t and the strain in the tangential direction is

$$\epsilon = \frac{\partial u_t}{\partial s} \tag{3}$$

The normal strain, ϵ is small and higher order terms are neglected.

B. *Strain-displacement matrices*

Consider a three node, iso-parametric cable element. The coordinates and displacements are approximated by using the nodal coordinate vector ($\boldsymbol{X}^a$) and nodal displacement vector ($\boldsymbol{d}^a$) in the following manner with Einstein summation scheme:

$$\boldsymbol{x} = N_a \boldsymbol{X}^a \tag{4}$$

$$\boldsymbol{u} = N_a \boldsymbol{d}^a$$

Node numbers are given by $a = 1,2,3$.

The tangential component of the displacement vector given in the previous section can be written in the following form.

$$u_t = \boldsymbol{u} \cdot \boldsymbol{t} \tag{5}$$

The normal strain can now be written as follows:

$$\epsilon = \frac{\partial \boldsymbol{u}}{\partial s} \cdot \boldsymbol{t} + \boldsymbol{u} \cdot \frac{\partial \boldsymbol{t}}{\partial s} \tag{6}$$

The arc-length derivative of displacement vector can be given in the standard form: