

Applicability of the Limit Equilibrium Method and the Finite Element Method in Predicting the Stability of Embankment Slopes

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Abstract: Evaluation of the stability of the embankment slopes is vital to ensure a safe and operational embankment. Stability of embankment slopes can be evaluated by using the available analytical methods or by field monitoring data. The most widely used analytical techniques include the limit equilibrium approach and the finite element approach. This study was carried out to investigate the applicability of the limit equilibrium approach and the finite element approach in predicting the stability of embankment slopes. Two test embankments published in the literature and three embankments, which belong to the Colombo- Katunayaka highway were analyzed by using the finite element method and the limit equilibrium method and the results have been compared with the field monitoring data. Results of the present study verify that both the finite element method and the limit equilibrium method can predict the stability of the embankment slopes reasonably well. In addition, the finite element method can be used to reasonably estimate the deformations of the embankments as well.

Keywords: Embankment stability, Finite element method, Limit equilibrium method

1. Introduction

Embankments are widely used in the construction of highways, railways, dams and retention dikes, harbor installations and airports. Today in Sri Lanka lands underlain by soft, weak and problematic soil are being utilized for various construction activities due to lack of suitable land. Design, construction and maintenance of an embankment over subsoil with low shear strength and high compressibility is an engineering challenge due to associated low bearing capacity and the excessive settlement. Therefore, it is vital to evaluate the stability of the embankment slopes during and after the construction to ensure a safe and operational embankment.

Stability of the embankment slopes can be evaluated by using available field monitoring data or analytical methods such as Finite Element Method (FEM) and Limit Equilibrium Method (LEM). The finite element analysis of geotechnical problems relies on the discretization of a continuum into a number of elements which are connected at nodal points. In FEM, stability of the embankment slopes can be done by using phi - c reduction method. Here strength parameters (Φ and c) of the soil are successively reduced until failure of the structure occurs. Therefore, factor of safety is defined as the ratio between the initial

strength parameter and the critical strength parameter.

In limit equilibrium method appropriate trial failure surfaces are assumed and the factor of safety values for each trial failure surface is determined. Minimum value among the above is taken as the factor of safety for the slope. Corresponding failure surface is the critical failure surface.

By using FEM both FOS and the deformation characteristics can be determined. However, LEM doesn't give any idea about the stress and deformation within the soil mass and it neglects the soil stress-strain relationship. However, despite the advantages of FEM over LEM, LE approach is still widely used in the evaluation of stability of slopes.

In view of the above, the main objective of the present study is to investigate the applicability of the Limit Equilibrium approach and the Finite Element approach in predicting the stability of embankment slopes with the help of several case histories published in the literature and monitoring data available in several local embankments.

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2. Background

Limited number of researches had been carried out to investigate the applicability of the LEM and FEM. Alkasawneh et al. (2007) analyzed two embankments using FEM and LEM and results obtained from the both of the programs are much similar. However they stated that LEMs are reliable and can be used with confidence to investigate the stability of slopes [1].

Maula & Zhang (2011) evaluated the stability of the homogenous soil slope by using FEM and the LEM by varying the shear strength parameters (c & Φ). For most cases, results given from the LEM and FEM are very similar [4].

Aziz (2010) evaluated the deformation and stability criteria of a reinforced embankment on soft clay using FEM and LEM. Factor of safety in FEM is smaller compared to the factor of safety obtained from LEM. However, predicted deformation values are different from the observed values. Aziz mentioned that this is probably due to the inaccuracies of the soil properties used for modelling [6].

Gunduz (2010) constructed a test embankment at the Lilla Mellösa and Skå-Edeby in Stockholm and the deformation characteristics of the embankment were evaluated by using FEM. It was reported that predicted data are much similar to the field measurements for both drained and un-drained conditions [7].

Huang et al. (2006) numerically analyzed the consolidation behaviour of an instrumented embankment on a soft soil foundation by using a coupled, nonlinear, finite element analysis. Predictions were compared with the field data. Predicted values for the settlement match well with the observed data. However, numerical model overestimates the horizontal displacement and the pore water pressure [2].

3. Methodology

Factor of safety values and the deformation characteristics of Aikoembankment (Shoji & Matsumoto, 1976) [5], the Muar embankment (Indraratna et.al, 1992) [3] and three slopes of the embankment at Colombo-Katunayake Expressway project (CKE) were analyzed by using finite element method and the limit equilibrium method and compared with the field observation data.

Finite element analysis was conducted by using PLAXIS 8.2 and plain strain 15 node elements were used and standard fixity option was used to define the boundary conditions. Embankment was analyzed by using updated mesh analysis. Factor of safety values were obtained by using Phi - c reduction technique and the vertical settlement at the centre of the embankment (d) and the lateral displacement at the toe of the embankment (δ) were obtained.

SLOPE/W 6.0 software was used to conduct Limit equilibrium analysis and adopted method for the analysis was Morgenstern and Price method.

3.1 Aiko test embankment

Shoji and Matsumoto (1976) constructed the Aiko test embankment in the Aiko district in the middle of the northern Kanagawa prefecture, Japan. Sub soil profile of the Aiko embankment is shown in Figure 1. Soil properties of the sub surface strata are presented in Table 2.

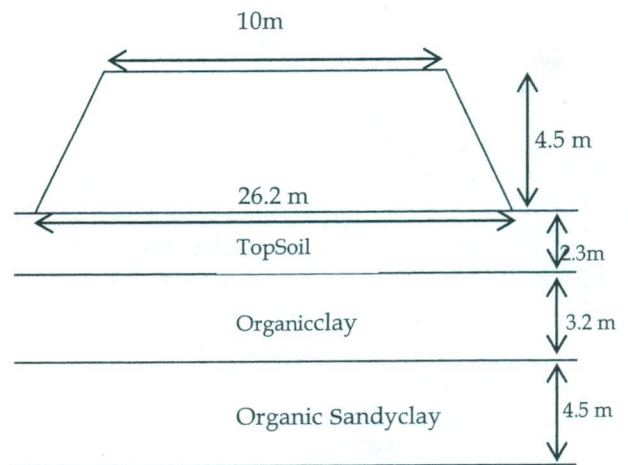


Figure 1 - Sub surface profile of the Aiko test embankment

Embankment was filled as one step within 18 days followed by 100 days consolidation period. In the present study, Aiko embankment was analyzed by using Finite Element Method and the Limit Equilibrium Method. In FEM, Soft Soil (SS) model was used to analyse the sub soil and basic parameters such as compression index (C_c), recompression index (C_r), initial void ratio, cohesion, friction angle, dilatancy angle and vertical and horizontal permeability values were used as input parameters. Mohr Coulomb (MC) model was used to represent the fill material and Young's modulus, Poisson's ratio, cohesion, friction angle and dilatancy angle were used as basic parameters.

3.2 Muar Test embankment

Indraratna et.al (1992) built this full scale test embankment on soft Malaysian marine clay. In the current study it was reanalyzed by using finite element method and limit equilibrium method and predicted values were compared with the field observation data. Geometry of the embankment is shown in Figure 2. Soil properties are given in Table 3. Soft Soil Creep (SSC) model was used to simulate the sub soil. Parameters used are similar to the SS model but coefficient of secondary consolidation ($C\alpha$) was used as an additional parameter.

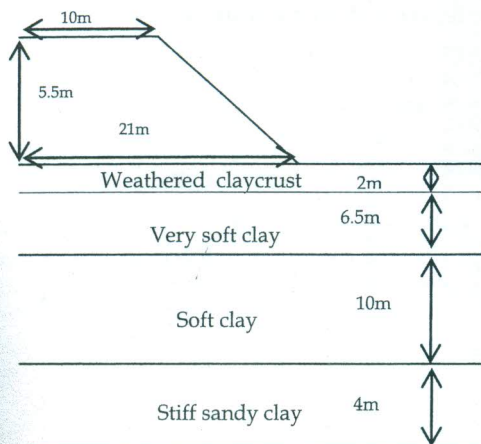


Figure 2 - Sub Surface profile of the Muar test embankment

Filling was done in three layers and filling sequence is shown in the Table 1.

Table 1 - Filling sequence of the Muar test embankment

Layer no	Fill height (m)	Time period (days)
I	2	35
II	2	35
III	1.5	26

3.3 Analysis of embankments at CKE

Three embankments (k7+870, k6+530 and k6+850), which belong to the Colombo-Katunayaka Expressway were analysed by using finite element method and limit equilibrium method. Geometries of the embankments are shown in Figures 3, 4 and 5 and filling sequences are shown in Table 5. Soil properties are given in Table 4. SS model and MC model were used to evaluate the sub soil.

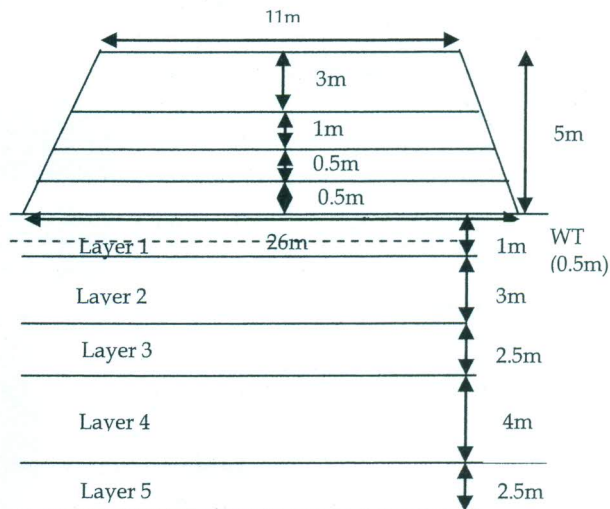


Figure 3- Details of the K6+850 embankment

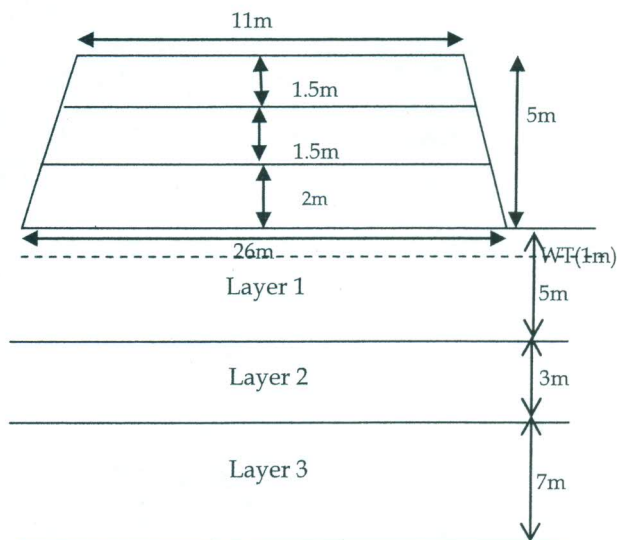


Figure 4 - Details of the K7+870 embankment

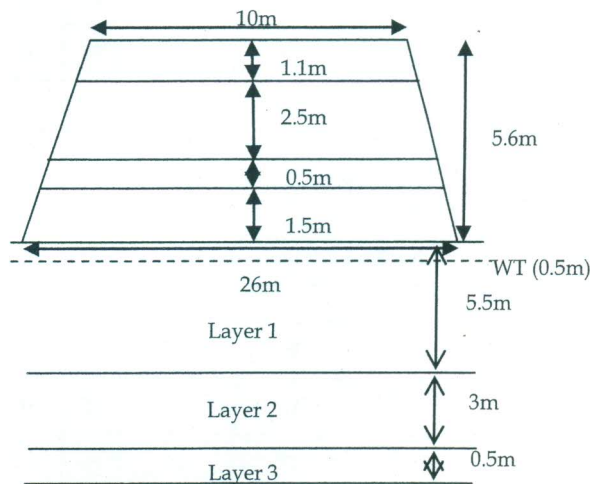


Figure 5 - Details of the K6+530 embankment

Table 2 - Soil properties of the Aiko test embankment

Layer / Soil type	γ_{unsat} kN/m ³	γ_{sat} kN/m ³	Φ	C_u kPa	k_y m/day	k_x m/day	E kPa	ν	C_c	C_r	e_{int}	Model	Condition
1/ Top soil	11	12	0.1	20	1.8*10 ⁻³	1.8*10 ⁻³	600	0.33	0.1042	0.0104	0.5	SS	Undrained
2/ Organic Clay	11	11	0.1	15	0.4	1.5	200	0.33	0.4213	0.0421	0.5	SS	Undrained
3/ Organic Sandy clay	11	13.5	0.1	25	0.02	0.02	850	0.33	0.1042	0.0104	0.5	SS	Undrained
fill	16	20	30	5	1	1	3000	0.3	-	-	-	MC	Drained

Table 3 - Soil properties of the Muar test embankment

Layer / Soil type	γ_{unsat} kN/m ³	γ_{sat} kN/m ³	Φ	C_u kPa	k_y m/day	k_x m/day	E kPa	ν	C_c	C_r	C_α	e_{int}	Model	Condition
1 / Weathered crust	16.5	17	0.1	15.4	6.912*10 ⁻⁵	1.296*10 ⁻⁴	-	0.3	0.75	0.075	0.05	0.5	SSC	Undrained
2/Very soft clay	15.5	16	0.1	13.4	6.912*10 ⁻⁵	1.296*10 ⁻⁴	-	0.4	0.75	0.075	0.05	0.5	SSC	Undrained
3/Soft clay	15.5	16	0.1	19.5	5.184*10 ⁻⁵	0.095*10 ⁻³	-	0.4	0.45	0.045	0.03	0.5	SSC	Undrained
4/Stiff sandy clay	16	17	0.1	25.9	5.184*10 ⁻⁵	0.095*10 ⁻³	-	0.3	0.15	0.015	0.006	0.5	SSC	Undrained
Fill/Lateritic soil	18	20	26	19	1	1	5100	0.3	-	-	-	-	MC	Drained

Table 4 - Soil properties of the K6+850, K7+870 and K6+530

K6+850														
Layer	γ_{unsat} kN/m ³	γ_{sat} kN/m ³	Φ	C_u kPa	k_y m/day	k_x m/day	E kPa	ν	C_c	C_r	e_{int}	Model	Condition	
Layer 1	17	18	28	0.1	86	86	15400	0.3	-	-	-	MC	Drained	
Layer2	16	16.5	0.1	26	1.8*10 ⁻⁴	9*10 ⁻⁵	3040	0.35	0.88	0.088	0.5	SS	Undrained	
Layer3	15	16	25	6	0.86	0.86	3880	0.33				MC	Drained	
Layer 4	16	16.5	0.1	26	1.8*10 ⁻⁴	9*10 ⁻⁵	3040	0.35	0.88	0.088	0.5	SS	Undrained	
Layer 5	15	16	25	6	0.86	0.86	3880	0.33				MC	Drained	
K7+870														
Layer 1	17	18	28	0.1	86	86	15400	0.3	-	-	-	MC	Drained	
Layer2	16	16.5	0.1	26	1.8*10 ⁻⁴	9*10 ⁻⁵	4000	0.35	0.73	0.073	0.5	SS	Undrained	
Layer3	16	17	25	6	9*10 ⁻³	9*10 ⁻³	12600	0.33	-	-	-	MC	Drained	
K6+530														
Layer 1	16	16.5	0.1	28	1.8*10 ⁻⁴	9*10 ⁻⁵	4000	0.35	0.88	0.088	0.5	SS	Undrained	
Layer2	16	17	25	10	9*10 ⁻³	9*10 ⁻³	12600	0.33	-	-	-	MC	Drained	
Layer3	17	18	30	0.1	86	86	15400	0.3	-	-	-	MC	Drained	
Fill	16	20	30	5	1	1	3000	0.3	-	-	-	MC	Drained	

Table 5 - Filling sequence of the K6+850, K7+870 and K6+530

K6 + 850			
Layer (Fill)	Height (m)	Construction period (days)	Consolidation period (days)
1	0.5	7	8
2	0.5	7	293
3	1	3	18
4	3	23	67
K7+870			
1	2	16	39
2	1.5	22	173
3	1.5	7	481
K6+530			
1	1.5	8	30
2	0.5	4	8
3	2.5	10	35
4	1.1	2	65

4. Results and Discussion

4.1 Results of Aiko Test Embankment

Results of the numerical analysis (lateral deformation at the toe of the embankment (δ) and the vertical settlement at the centre of the embankment (d)) and field monitoring data are given in Table 6.

Table 6 - Deformation values for the Aiko embankment

Predicted values		Observed values	
d (m)	δ (m)	d (m)	δ (m)
2.616	1.308	1.5	0.2
Updated mesh analysis			
1.549	0.356		

According to the manual calculations,

Primary consolidation settlement at the center of the embankment = 1.35m

Secondary consolidation settlement at the center of the embankment = 0.174m

Total settlement = 1.524m

According to the above results, there's a good agreement between the predicted values and the observed values and they are similar to the values obtained from the manual calculations. On the other hand, in finite element analysis, updated mesh analysis gives lower deformation values. Generally soft soils incorporate with larger deformations therefore it is necessary to consider about the influence of the geometry changes of the mesh on the equilibrium condition. As such the values given

by the updated mesh analysis are more reliable than the conventional finite element analysis. Because updates mesh analysis includes second order deformations and it takes account of the changes of geometry.

Factor of safety value obtained from the finite element method is 1.263 and the limit equilibrium method gives 1.401.

It can be observed that the Factor of safety value obtained from the finite element method is slightly lower than that obtained from limit equilibrium method.

4.2 Results of Muar Test embankment

Lateral displacement at a location 10m away from the center line of the embankment was interpreted by using FEM as shown in Figure 6. Field data were reproduced by using original work done by Indraratne et al. (1992).

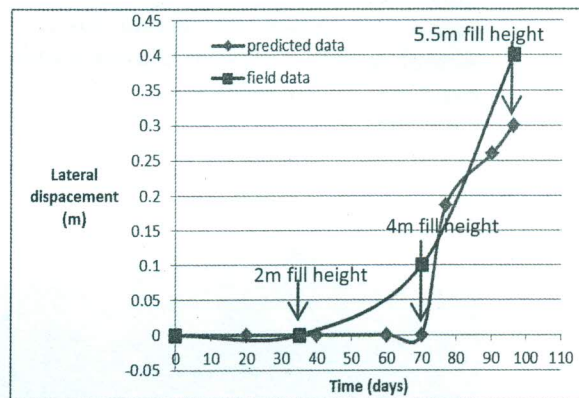


Figure 6 -Variation of lateral displacement with time - Muar embankment

Variation of the settlement at the centre of the embankment with time was plotted and shown in the Figure 7. Indraratne et al. (2005) found that the variation of the surface settlement for the embankment of height 5m as given in Figure 8. Predicted value for the settlement is 0.68m and observed value is equal to 0.6m (refer to the value of y axis when distance from the centre is zero at Figure 8).

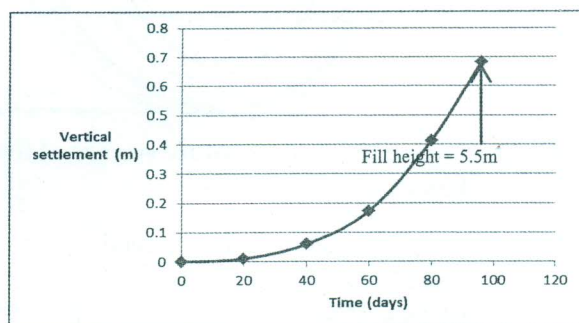


Figure 7 - Variation of settlement with time - Muar embankment

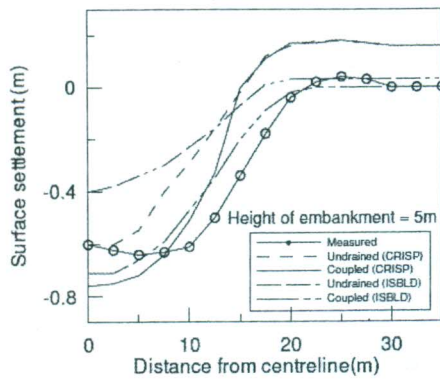


Figure 8 - Surface settlement profiles for 5m fill height

According to the above figures, predicted values for the lateral displacement and vertical settlement match quite well with the observed values.

Predicted Failure surface obtained from the finite element analysis is shown in the Figure 9. Figure 10 shows the actual failure surface. Actual failure surface is almost similar to the predicted failure surface.

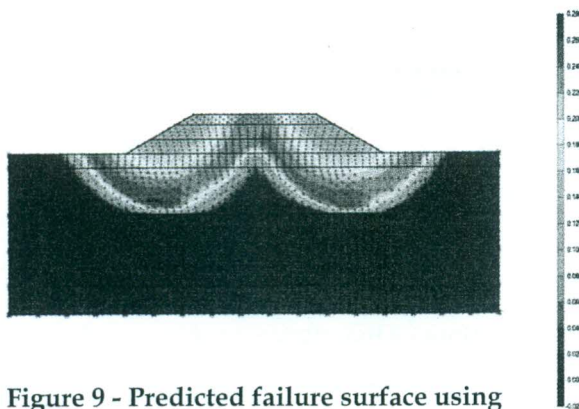


Figure 9 - Predicted failure surface using FEM for Muar test embankment

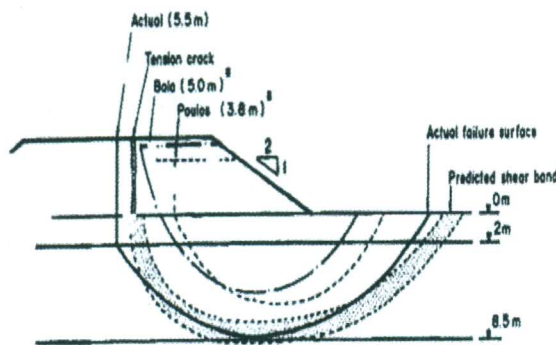


Figure 10 - Actual failure surface of the Muar test embankment

Factor of safety value obtained from the FEM is 0.826 and LEM gave 0.926. Predicted failure surface obtained from the limit equilibrium method is shown in Figure 11.

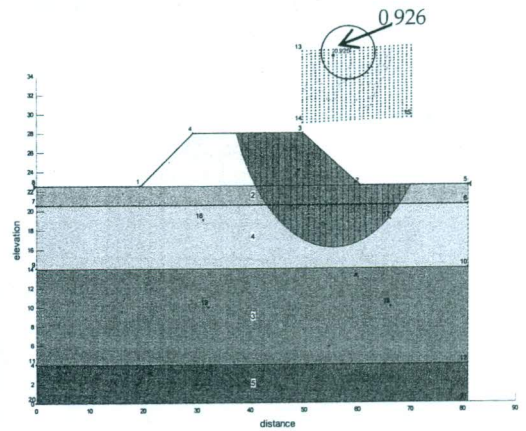


Figure 11 - Predicted failure surface using LEM for Muar test embankment

It can be observed from the results that the predicted failure surface obtained from the LEM method is slightly higher than that obtained from FEM.

4.3 Results of the actual embankments

Predicted and the observed deformation values are shown in the Table 7.

Table 7 - Predicted and observed deformations of CKE embankments

Embankment No	Predicted values		Observed values	
	d(m)	δ (m)	d(m)	δ (m)
K7+870	0.489	0.022	0.968	0.819
K6+850	0.763	0.059	0.666	0.089
K6+530	0.790	0.091	0.116	0.034

Factor of safety values obtained from the FEM and the LEM method are given in the Table 8.

Table 8 - FOS values of the CKE embankments

Embankment no	FOS	
	LEM	FEM
K6+ 530	1.746	1.711
K6+850	1.739	1.411
K7+870	1.612	1.487

Among these three embankments, K7+870 is a failed embankment. However, LEM and the FEM give larger factor of safety values and they don't reflect this instability condition. This may be due to the inaccuracies of the soil properties used for modelling or due to poor workmanship during the construction of the embankment that caused the failure. Soil properties used were obtained by using cone penetration test results and empirical formulas. However, Factor of Safety values obtained for

the embankments using FEM are slightly lower than those obtained from LEM.

5. Conclusions

According to the deformation values obtained for the Aiko embankment there's a good agreement between the predicted, observed and calculated values. Similarly for Muar test embankment, predicted deformation values using FEM are much similar to the observed values.

Furthermore, according to the results of stability analysis of Aiko, Muar and CKE embankments, it can be seen that the Factor of safety values obtained from the finite element method are slightly lower than those obtained from limit equilibrium method.

Critical failure surface can be accurately traced by the FE analysis.

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