Inverse Modelling for Identifying the Origin and Release rate of Atmospheric Pollution -An Optimisation Approach

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Abstract: This paper describes a mathematical modelling technique which may be used to identify the origin and release history of a polluting gas released into the atmosphere from a point source. The inputs to this model are: pollution concentration measurements made at ground locations downstream, wind speed and transport parameters. The inverse model is formulated as a non-linear least-squares minimisation problem coupled with the solution of an advection-dispersion equation for a non-steady point source. The minimisation problem is ill-posed; consequently its solution is extremely sensitive to errors in the measurement data. Tikhonov's regularisation, which stabilises the solution process, is used to overcome the ill-posedness. Since the minimisation problem has a combination of linear and non-linear parameters, the problem is solved in two steps. Non-linear parameters are found by constructing an iterative procedure and, at each iteration, the linear parameters are calculated. The optimal value of the regularisation parameter is obtained by incorporating the L-curve criterion from linear inverse theory in conjunction with maintaining a steady increase in the regularisation parameter from one iteration to the next. Finally, the accuracy of the model is examined by imposing a normally-distributed relative noise into concentration data generated by the forward model.

Keywords: *Non-linear ill posed problem; Inverse air pollution model; Parameter estimation*

1. INTRODUCTION

Atmospheric dispersion modelling may be used in a post-accident management plan to evaluate conditions in the case where the accidents involve gas leakages. Atmospheric dispersion models describe the transport and dispersal of pollutant gases in the atmosphere. A dispersion model, which is capable of describing the behaviour of air pollutants in the atmosphere, requires the following input data: (1) meteorological data such as wind speed, direction and atmospheric stability, and (2) the source emission rate and its origin. In reality, meteorological data can be measured using available measuring instruments, but the source release rate and its origin are often unknown. Methods to determine the release rate and the origin of the source of the pollutant gas are therefore a significant part of modelling atmospheric dispersion.

The procedure for identifying the origin and release rate of a gas from observations of pollutant concentration reduces to a parameter estimation problem in an air pollution model. Several articles (Edwards *et al.*, 1993; Kibler & Suttles, 1977; Sohier *et al.*, 1997), have been published in this area. In these approaches air pollution transport models for the steady-state point source are used as representative of the pollution transport process, but none of these are based on the advection-dispersion model.

A variety of numerical and analytical techniques have been proposed (Skaggs & Kabala, 1994; Woodbury & Ulrych, 1996; Neupauer *et al*, 2000) to solve similar problems in the area of groundwater modelling. Because of the physical and mathematical similarities between the mass transfer in water and air, mathematical techniques used in groundwater modelling are also relevant to the problem of air pollution modelling.

The novel concept of this study is to identify the origin and release history of a polluting gas based on methods available in the groundwater modelling literature. In one of our previous papers (Kathirgamanathan *et al.*, 2002) we addressed the same problem. In this paper we report a different technique as a computation-