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Some results on G-frame operator in quaternionic setting

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G frames are a natural generalization of frames which cover many other extensions of frames. Quaternions are an extension of complex numbers from the two-dimensional plane to four-dimensional space and form a non-commutative associative algebra. Due to the non-commutativity, there are two types of Hilbert spaces over quaternions, called right quaternionic Hilbert space and left quaternionic Hilbert space. In this research G-frame operator for G-frame in left quaternionic Hilbert space $V_{\mathbb{H}}^{L}$ is introduced and some results of G-frame operator are presented and one can easily obtain these results on right quaternionic Hilbert space $V_{\mathbb{H}}^{R}$ by the symmetry. Let $U_{\mathbb{H}}^{L}$ and $V_{\mathbb{H}}^{L}$ be two left quaternionic Hilbert spaces and $\{\mathcal{V}_k : k \in \mathbb{I}\} \subseteq V_{\mathbb{H}}^{L}$ is a sequence of quaternionic Hilbert spaces. A family of sequence $\{\Lambda_k \in \mathcal{B}(U_{\mathbb{H}}^{L}, \mathcal{V}_k) : k \in \mathbb{I}\}$ is called generalized frame or simply G-frame for $U_{\mathbb{H}}^{L}$ with respect to $\{\mathcal{V}_k : k \in \mathbb{I}\}$ if there exist constants $0 < A \leq B < \infty$ such that $A\|f\|^2 \leq \sum_{k\in\mathbb{I}} \|\Lambda_k f\|^2 \leq B\|f\|^2$, for all $f \in U_{\mathbb{H}}^{L}$, where A and B are G-frame bounds. We call $\{\Lambda_k \in \mathcal{B}(U_{\mathbb{H}}^{L}, \mathcal{V}_k) : k \in \mathbb{I}\}$ is a tight G-frame if $A = B$. If $\{\Lambda_k\}_{k\in\mathbb{I}}$ is a G-frame in $U_{\mathbb{H}}^{L}$ with G-frame operator S^G if and only if $AI_{op} \leq S^G \leq BI_{op}$ and $\{\Lambda_k\}_{k\in\mathbb{I}}$ is G-normalized tight frame in $U_{\mathbb{H}}^{L}$ if and only if $S^G = I_{op}$, where I_{op} is an identity operator in $U_{\mathbb{H}}^{L}$. If S^G is a G-frame operator for the G-frame $\{\Lambda_k\}_{k\in\mathbb{I}}$ with frame bounds A and B in $U_{\mathbb{H}}^{L}$ then $B^{-1}I_{op} \leq S^{G^{-1}} \leq A^{-1}I_{op}$. Finally sequence of operator $\{\widetilde{\Lambda_k}\}_{k\in\mathbb{I}}$ (where $\widetilde{\Lambda_k}=\Lambda_k S^{G^{-1}}$) is G-frame for the quaternionic Hilbert space $U_{\mathbb{H}}^{L}$ with frame bounds $\frac{1}{B}$ and $\frac{1}{A}$. We have seen that a sequence of operators is a G-frame for the left quaternionic Hilbert space $V_{\mathbb{H}}^{L}$ with frame bounds $\frac{1}{B}$ and $\frac{1}{A}$.

Keywords: *Frames, G-frames, Quaternionic Hilbert space.*