Identification of Two Groups with the Rapid Decay Property

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Abstract: Rapid decay property (property (RD)) for groups, generalizes Haagerup's inequality for free groups and so for example of free groups have property RD. Property RD provides estimates for the operator norm of those functions (in the left-regular representation) in terms of the Sobolev norm. Even more, property RD is the noncommutative analogue of the fact that smooth functions are continuous. This property RD for groups has deep implications for the analytical, topological and geometric aspects of groups. It has been proved that groups of polynomial growth and classical hyperbolic groups have property RD, and the only amenable discrete groups that have property RD are groups of polynomial growth. He also showed that many groups, for instance $SL_3(\mathbb{Z})$, do not have the Rapid Decay property. Examples of RD groups include group acting on CAT(0)-cube complexes, hyperbolic groups of Gromov, Coxeter groups, and torus knot groups. The symmetry group of a tiling pattern of the plane is called a *crystallographic group*. The discrete *Heisenberg group* is the multiplicative group H_3 of all matrices of the form

$$\begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$$

where $a, b, c \in \mathbb{Z}$.

The sufficient conditions on property RD for extensions to have property RD with respect to the word length. The following important result, which is used for the main result of this paper: Let G and Γ be two discrete groups of finite type, and $\{e\} \to G \xrightarrow{i} E \xrightarrow{\pi} \Gamma \to \{e\}$ a split extension of G by Γ. If G (respectively Γ) is finite, then E has property RD with respect to the word length if and only if Γ (respectively G) has property RD with respect to the word length. These papers provide a simple method to establish the Crystallographic groups and the Discrete Heisenberg group having property RD.

Keywords: Property RD, The Crystallographic groups, The Discrete Heisenberg Group