Coherent States for an Abstract Hamiltonian with a General Spectrum

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Abstract: Following the method proposed by Gazeau and Klauder to construct temporally stable coherent states, CS for short, in recent years, several classes of CS were constructed for quantum Hamiltonians. The spectrum $E(n)$ of several solvable quantum Hamiltonians is a polynomial of the label n. In this letter, we discuss CS with a general spectrum $E(n) = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$, of degree k, which is considered as the spectrum of an abtract Hamiltonian. As special cases of our construction we obtain CS for the quantum Hamiltonians, namely; Harmonic oscillator, Isotonic oscillator, Pseudoharmonic oscillator, Infite well potential, Pöschl-Teller potential and Eckart potential. We shall also exploit the coherent states on a letf quaternionic separable Hilbert space with the spectrum $E(n)$. Let us introduce the general features of Gazeau-Klauder CS. Let H be a Hamiltonian with a bounded below discrete spectrum $\{e_n\}_{n=0}^{\infty}$ and it has been adjusted so that $H \geq 0$. Further assume that the eigenvalues e_n are non-degenerate and arranged in increasing order, $e_0 < e_1 < \cdots$. For such a Hamiltonian, the so-called *Gazeau-Klauder coherent states* (GKCS for short) are defined as

(0.1) $$|J, \alpha\rangle = \mathcal{N}(J)^{-1} \sum_{n=0}^{\infty} \frac{J^{n/2}}{\sqrt{\mathcal{K}(n)}} e^{-ie_n \alpha} \eta_n$$

where $J \geq 0, -\infty \leq \alpha \leq \infty, \{\eta_n\}_{n=0}^{\infty}$ is the set of eigenfunctions of the Hamiltonian and $\mathcal{K}(n) = e_1 e_2 \ldots e_n = e_n!$. In order to be GKCS the states (0.1) need to satisfy the following:

a) For each J, α the state is normalised, i.e., $\langle J, \alpha | J, \alpha \rangle = 1$;
b) The set of states $\{|J, \alpha\rangle : J \in [0, \infty), \alpha \in (-\infty, \infty)\}$ satisfies a resolution of the identity
$$\int_0^{\infty} \int_{-\infty}^{\infty} |J, \alpha\rangle\langle J, \alpha| d\mu(J, \alpha) = I$$
where $d\mu(J, \alpha)$ is an appropriate measure;
c) The states are temporally stable, i.e., $e^{-i\omega Ht} |J, \alpha\rangle = |J, \alpha + \omega t\rangle$;
d) The states satisfy the action identity, i.e., $\langle J, \alpha | H | J, \alpha \rangle = J$.

The condition (d) requires $e_0 = 0$. In the case where only the conditions (a)–(c) are satisfied the resulting CS may be phrased as "temporally stable CS".

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