Asset pricing with liquidity risk: Evidence from a Frontier Market
Herath, H. M. K. M\textsuperscript{a} and Samarakoon, S. M. R. K\textsuperscript{b}

\textsuperscript{a,b}Department of Accountancy, Faculty of Business Studies and Finance, Wayamba University of Sri Lanka, Sri Lanka

\textsuperscript{a}madusanka.sherath@gmail.com

Abstract
This study aims to analyze the relationship between stock returns and liquidity risk while taking into account the time-varying characteristics of illiquidity on the Colombo Stock Exchange from 2015-2019 and taking into account the effect of liquidity level, using the Generalized Method of Movements (GMM) framework model to assess the persistence of illiquidity stocks. The updated version of Amihud Illiquidity (Amihud, 1986) is a contribution that represents illiquidity and research across the time-series relationship between liquidity and return. The price of liquidity risk and its effect on expected returns are tested empirically using the conditional liquidity adjusted capital asset pricing model (LCAPM), where stock returns are cross-sectionally dependent on market risk and three additional betas ($\beta_1, \beta_2, \beta_3$) that capture different aspects of illiquidity and its risk. The results show some support for the conditional capital asset pricing model (CAPM), but the results are not robust to alternative requirements and estimation methods. The total effect of liquidity risk is 0.11\%, and illiquidity is 2.5\% per year. The total annualized illiquidity premium is therefore found that 2.61\% in the Colombo stock exchange.

Keywords: asset pricing, Colombo Stock Exchange, generalized method of movements and liquidity risk

Introduction
Liquidity has a significant role in financial markets by improving the sharing of risk and improving trade efficiency. Early literature on the impact on asset prices of liquidity rates indicates that illiquid stocks attract a price premium. A more recent section of the literature examines the connection between liquidity risk and returns of assets. This study aims to examine how liquidity risks and common liquidity are influenced by the liquidity adjusted asset pricing model (LCAPM), with special illiquidity proxies, in which the market effects are also correlated to the probabilities of variations in individual securities liquidity. Based on the background, the current study aims to fill a

\footnotetext{1}{In this study, it will use liquidity and illiquidity interchangeably. Both terms infer that an investor should receive a premium for the associated risk of holding assets with illiquidity cost and risk.}
gap in the existing literature by addressing the following research problem as, "How asset prices are affected by liquidity risk and commonality in liquidity". If liquidity is persistent, an investor can estimate tomorrow's liquidity level with the data of today. Therefore, it is crucial to estimate if the liquidity level is persistent instead of assuming that persistence exists. According to the objective of the study, research questions are as follows:

- Is there any relationship between liquidity risk and asset pricing?
- How does liquidity risk affect asset prices in equilibrium?
- How do the effects of liquidity risk vary in different market states?

**Literature Review**

It is estimated in the literature that the level of liquidity is priced. Acharya and Pedersen (2005) found that the liquidity risk was positively priced using the Amihud liquidity measure (ALM) and the liquidity-based asset pricing model. Amihud and Mendelson, (1986) were the first to examine the relationship between liquidity, asset prices, and how this is interlinked with investors holding period, and found that investors trading more often would prefer to hold assets with lower transaction costs. Brennan et al., (1998) examine the relationship between the illiquidity premium and returns while measuring the alternative liquidity proxy that measures price impact and market depth. Jones (2001) finds evidence that the expected returns are the same when the spread is large. While using the turnover ratio as a measure of liquidity, he finds that a high turnover ratio leads to lower returns on stocks. Using daily data Hasbrouck and Seppi (2001) get mixed results. He finds that the relationship between returns and liquidity significantly varies in scope and direction. Kumar and Misra (2019) results suggest that liquidity forms part of the systematic and idiosyncratic risk.

Therefore, failure to incorporate it into portfolio formulation strategies may lead investors on the National Stock Exchange (NSE) to make erroneous investment decisions. Liquidity is a multidimensional concept. Hence, the studies on liquidity usually consider multiple liquidity measures in research. Using Amihud’s illiquidity measure as the only measure in modelling asset pricing is one of this study's limitations.

**Methodology**

The data used in this study are collected from several sources—daily frequency data on all common stocks available on CSE. The data set used in the study covers the period from 2015 to 2019 and in the population of this study includes all the companies listed in the Colombo stock exchange in Sri Lanka, which does not include investments, financial intermediaries, banks,
insurance companies, private equity, real estates. Fifty companies were selected under a stratified random sampling method. The data set includes information on the return on the market and firms, market capitalization, turnover, and the risk-free rate. Only ordinary shares are included in the selection, and these are adjusted for dividends—the empirical work on an equal-weighted market portfolio.

Portfolio Construction
It is formed illiquidity portfolios during the period 2015-2019 for each year. Firstly, at the beginning of each year, build ten illiquidity portfolios based on daily illiquidity calculations by employed daily return and volume data from previous years. The primary test is defined in terms of equally-weighted returns and illiquidity for the portfolio of markets.

Illiquidity measure
There are several liquidity measures and proxies; this research uses the (Amihud, 2002) illiquidity factor. The ALM is a measure of illiquidity because it measures the price impact of trading in percentage; a higher outcome hints at a higher level of illiquidity. The formula for the ALM is presented as follows:

\[
ILLIQ_t^i = \frac{1}{\text{Days}_t} \sum_{d=1}^{\text{Days}_t} \frac{R_{td}^i}{V_{td}^i}
\]

…Equation 1

Source: Amihudand Mendelson(1986)

Unconditional LCAPM
The researcher calculates an unconditional model to estimate the liquidity-adjusted CAPM. For example, under the conditions of independence over time of dividends and liquidity costs, and the unconditional outcome is obtained. Nonetheless, it is empirically continuous. The researcher is, therefore, based on the assumption that developments in illiquidity and returns are constantly conditional. This assumption yields the unconditional result that,

\[
E(r_t^p - r_t^f) = E(C_t^p) + \lambda \beta_1^p + \lambda \beta_2^p - \lambda \beta_3^p - \lambda \beta_4^p
\]

…Equation 2

Where,

\[
\lambda = E(\lambda_t) = E(r_t^M - c_t^M - r_t^f)
\]
\[ \beta^{1p} = \frac{cov(r_t^{M} - E_t-1(r_t^{M})), r_{t-1}^{M} - E_t-1(c_t^{M})}{var((r_t^{M} - E_t-1(r_t^{M})), -[c_t^{M} - E_{t-1}(c_t^{M})])} \]  
Equation 3

\[ \beta^{2p} = \frac{cov(c_t^{M} - E_t-1(c_t^{M})), c_{t-1}^{M} - E_t-1(c_t^{M})}{var((r_t^{M} - E_t-1(r_t^{M})), -[c_t^{M} - E_{t-1}(c_t^{M})])} \]  
Equation 4

\[ \beta^{3p} = \frac{cov(r_t^{M} - E_t-1(c_t^{M})), r_{t-1}^{M} - E_t-1(c_t^{M})}{var((r_t^{M} - E_t-1(r_t^{M})), -[c_t^{M} - E_{t-1}(c_t^{M})])} \]  
Equation 5

\[ \beta^{4p} = \frac{cov(c_t^{M} - E_t-1(c_t^{M})), r_{t-1}^{M} - E_t-1(c_t^{M})}{var((r_t^{M} - E_t-1(r_t^{M})), -[c_t^{M} - E_{t-1}(c_t^{M})])} \]  
Equation 6


\( \beta^{1p} \) - covariance between the return of a security and the market return.
\( \beta^{2p} \) - covariance between asset’s illiquidity of a stock and the market illiquidity.
\( \beta^{3p} \) - covariance between a security’s return and market liquidity.
\( \beta^{4p} \) - covariance between a security’s illiquidity and the market return.

Results and Discussions

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>( \beta^{1p} ) (100)</th>
<th>( \beta^{2p} ) (100)</th>
<th>( \beta^{3p} ) (100)</th>
<th>( \beta^{4p} ) (100)</th>
<th>( \beta^{Net.p} ) (100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.03</td>
<td>0.62</td>
<td>-0.24</td>
<td>-0.05</td>
<td>50.95</td>
</tr>
<tr>
<td>2</td>
<td>40.25</td>
<td>0.07</td>
<td>-0.26</td>
<td>-0.24</td>
<td>40.82</td>
</tr>
<tr>
<td>3</td>
<td>88.04</td>
<td>0.02</td>
<td>-0.36</td>
<td>-0.24</td>
<td>88.68</td>
</tr>
<tr>
<td>4</td>
<td>85.72</td>
<td>0.09</td>
<td>-0.52</td>
<td>-0.24</td>
<td>86.57</td>
</tr>
<tr>
<td>5</td>
<td>74.02</td>
<td>0.02</td>
<td>-0.54</td>
<td>-0.43</td>
<td>75.01</td>
</tr>
<tr>
<td>6</td>
<td>69.52</td>
<td>0.84</td>
<td>-0.67</td>
<td>-0.59</td>
<td>71.62</td>
</tr>
<tr>
<td>7</td>
<td>81.94</td>
<td>0.29</td>
<td>-1.18</td>
<td>-0.82</td>
<td>83.86</td>
</tr>
<tr>
<td>8</td>
<td>44.49</td>
<td>0.06</td>
<td>-1.45</td>
<td>-0.68</td>
<td>46.68</td>
</tr>
<tr>
<td>9</td>
<td>82.12</td>
<td>0.74</td>
<td>-1.48</td>
<td>-0.82</td>
<td>85.16</td>
</tr>
<tr>
<td>10</td>
<td>57.14</td>
<td>1.51</td>
<td>-1.78</td>
<td>-0.95</td>
<td>61.38</td>
</tr>
</tbody>
</table>

Looking at the market beta \( \beta^{1p} \) denotes the covariance between the return of a security and the market return. The market betas have a positive value linear with the required security return and have positive with the liquidity stocks, and its value is large. And also, liquidity betas, \( \beta^{2p} \) have different values in illiquidity, while \( \beta^{3p} \) start- off with a small negative value and the sign of \( \beta^{3p} \) varies from small negative values to large negative values in portfolios of each security and has an obscure pattern in the test portfolios. And also, there are no positive values these all testing portfolios \( \beta^{3p} \) value got negative.
values. If the researcher interprets $\beta^{3p}$ in an economic sense, the investors expect returns of the stocks in the liquid companies to remain stable in times of illiquidity in the market. Said differently, negative values of $\beta^{3p}$ for portfolio 1 to 10 means that the returns of the portfolios react too much to market illiquidity, i.e., high sensitivity of returns to market liquidity. The liquid stock (portfolio 10) seems to have a higher sensitivity of returns to market illiquidity. Conclusion this is interesting on its own, but since the most illiquid portfolios are not supported in terms of statistical significance, they should be careful in this consideration. And also, $\beta^{4p}$ is negative for all liquid securities, though this value is small and has an obscure pattern. $\beta^{4p}$ seem to be increasing slowly between portfolio 1 and 10. And these portfolios are most sensitive to market returns.

A few assumptions are needed to study the relationship between liquidity risk and expected returns, and some model constraints are set. To test this relation using the General Method of Moments (GMM) framework by carrying out a cross-sectional regression of portfolios. Running GMM generates similar estimates as the traditional cross-sectional regression or using pooled OLS, but GMM also enables serial correlation and takes into account the pre-estimation of betas. The application of GMM in empirical asset pricing is provided in Cochrane (2001).

According to that firstly, set a constraint that the risk premium for the betas is the same, defined as,

$$\beta^{netp} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$$

.................................................................................. **Equation 7**

Which makes liquidity adjusted CAPM:

$$E\left( r_{t}^{p} - r_{t}^{f} \right) = \alpha + kE\left( c_{t}^{p} \right) +$$

$$\beta^{netp} .................................................................**Equation 8**

Where the researcher allows a nonzero intercept, $\alpha$, even though Acharya and Pedersen (2005) claim that this intercept should zero.

Table 2. Asset Pricing: Model Testing for Illiquidity Sorted Portfolios

<table>
<thead>
<tr>
<th>Constant</th>
<th>$E(c^{p})$</th>
<th>$\beta^{1p}$</th>
<th>$\beta^{2p}$</th>
<th>$\beta^{3p}$</th>
<th>$\beta^{4p}$</th>
<th>$\beta^{netp}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.201***</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td>0.250**</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td>(-2.774)</td>
<td>(-)</td>
<td></td>
<td></td>
<td></td>
<td>(1.270)</td>
<td>(0.115)</td>
</tr>
<tr>
<td>2</td>
<td>-0.123</td>
<td>-0.01</td>
<td></td>
<td></td>
<td></td>
<td>0.702</td>
<td>0.225</td>
</tr>
<tr>
<td></td>
<td>(-1.155)</td>
<td>(-0.534)</td>
<td></td>
<td></td>
<td></td>
<td>(0.853)</td>
<td>(0.223)</td>
</tr>
<tr>
<td>3</td>
<td>-0.144**</td>
<td>0.164</td>
<td></td>
<td></td>
<td></td>
<td>0.123</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.201)</td>
<td>(0.155)</td>
<td></td>
<td></td>
<td></td>
<td>(0.121)</td>
<td></td>
</tr>
</tbody>
</table>
For specific configurations, the average holding period $k$ for illiquidity sorted portfolios is calibrated to 0.025. This implies that it takes $1/0.025 \approx 40$ months for all stocks to be turned over once, which corresponds to investors holding, and this value is obtained by the averaging turnover of test portfolios.

To isolate the effect of liquidity risk, $\beta^{2p}$, $\beta^{3p}$ and $\beta^{4p}$ over traditional market risk, $\beta^{1p}$, and liquidity level, $E(c^p)$, consider the following model,

$$E(r_t^p - r_t^f) = \alpha + kE(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p}$$

This relation is estimated with $k$ at its calibrated value. In this specification, $\beta^{net,p}$ is still significant (at 5% and 1%), but $\beta^{1p}$ seem to produce relatively small values while being significant. Equation (2), (5) and (8) produce quite different results when allowing $k$ to be free parameter and $\beta^{net,p}$ has different in value, while getting a small positive value of $\beta^{1p}$ and small increased value of $E(c^p)$. In equation (6), set $k = 0$, which leads to support for $\beta^{net,p}$. It is also worthy of note that the negative value of $\beta^{1p}$ in equation (5) and (6) does not mean a negative risk premium $\lambda^M$ in the market. Since it has included $\beta^{1p}$ as a part of $\beta^{net,p}$, simply need to add the coefficient of $\beta^{net,p}$ to get the correct value. For instance, in Equation (5) in Table 2 means that,

$$E(r_t^p - r_t^f) = 0.148 + 0.04E(c_t^p) - 0.260\beta^{1p} + 0.382\beta^{net,p}$$

$$E(r_t^p - r_t^f) = 0.148 + 0.04E(c_t^p) + 0.122\beta^{1p} + 0.382(\beta^{2p} - \beta^{3p} - \beta^{4p})$$

To test the full model, the researcher allows the betas to have different risk premiums and $\lambda$ and a fixed $k$, and run the unrestricted model obtained in equation (7). Equation (8) runs the same model with $k$ as a free value. Here is the generalized relation,

$$E(r_t^p - r_t^f) = \alpha + kE(c_t^p) + \lambda^1 \beta^{1p} + \lambda^2 \beta^{2p} + \lambda^3 \beta^{3p} + \lambda^4 \beta^{4p}$$
If there is no model restriction, $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4$. Also, see that all betas’ produce moderate results, both significant and insignificant, except for the average illiquidity portfolio, $E(c^P_t)$.

Since there is a significant collinearity problem, however, this evidence should be interpreted with caution. Eventually, it wants to emphasize that the intercept $\alpha$ fluctuates between being significant and insignificant of some specification, while the model implies a zero-constant value.

Then, the results’ economic significance and the overall liquidity risk is probably more important to research. The annual market risk premium should be measured to show the size of the results, $\lambda^M$, and the market risk premium for different liquidity betas (i.e. $\lambda^1, \lambda^2, \lambda^3, \lambda^4$) required to hold illiquid stocks. This calculates by the market risk premium product and the difference in empirical literature between liquidity risk for most liquid and least portfolio. The different annualized expected returns between portfolio 1 and 10 that can be attributed to a difference in $\beta^{2p}$.

Hence using the calibrate value $k$ and the common market risk premium, $\lambda^M$, of 0.250 from Equation 1 get the following results, the commonality of portfolio illiquidity and market illiquidity is, $\lambda^M(\beta^{P10}_2 - \beta^{P1}_2)12 = 0.026\%$

Similarly, the effect of $\beta^{3p}$, the sensitivity of returns to market illiquidity, on yearly returns is,

$-\lambda^M(\beta^{P10}_3 - \beta^{P1}_3)12 = 0.05\%$

And similarly, the effect of $\beta^{4p}$, the sensitivity of portfolio illiquidity to the overall market return is

$-\lambda^M(\beta^{P10}_4 - \beta^{P1}_4)12 = 0.03\%$

which makes the overall effect of liquidity risk of 0.11% per year.

Annualized expected rates of return between portfolio 1 and 10 are 2.5% difference based on the calibrate coefficient, attributed to an expected illiquidity difference, $E(c^P_t)$. The cumulative impact of the expected probability of illiquidity and liquidity is 2.61% per year.
In the restricted model, the overall liquidity risk defined as the liquidity beta with a single market risk premium is relatively low and barely significant. Innovations in liquidity are interpretable as liquidity shocks as economic crises. Figure 1 shows the standardized, normalized innovations in illiquidity; the innovations standardized by their standard deviation.

![Graph](image)

**Fig 2. Standardized innovation in market illiquidity from 2015-2019**

**Conclusions and Recommendations**
Finally, this empirical analysis suggests that the effects of liquidity and liquidity risk are separate. The traditional asset pricing model has been adjusted to reflect the cost of illiquidity and its respective risks over time. It is found that investors are interested in securities returns and illiquidity, especially in the downstream market. Investors' returns are positively affected by this liquidity and increase the co-variance between securities illiquidity and market-wide illiquidity. Returns are increasing if the stock performance is highly sensitive to market-wide illiquidity, and the co-variance between illiquidity and gross market returns is declining. Returns are also rising for most illiquidity stocks, characterized by small firm size and low turnover. The volatility of returns, however, appears to be the highest for the most liquid companies.

The findings are important for all market participants and require more research on diversifying the liquidity risk internationally. Investors should
also reconsider their liquidity investment strategy if they focus on achieving an above normal excess return by investing in assets with a high illiquidity level. Further research should determine whether the international version of the LCAPM yields a better result because of the international relationship in liquidity risk or the international commonality between stock return and market return. This thesis's results are based on an unconditional model, which implies that the risk aversion and the liquidity risk of investors are constant. This implies that the risk premiums are constant.

References


